

SX-5443

STREAM - A

ANDHRA UNIVERSITY
DEPARTMENT OF MATHEMATICS
M.A/M.SC MATHEMATICS
IV-SEMESTER

2005-2006 AB

M 403(1) NUMBER THEORY- II

UNIT-I :- FINITE ABELIAN GROUPS AND THEIR CHARACTERS:-

Characters of finite abelian groups- The character group- The orthogonality relations- for characters- Dirichlet characters- Sums involving Dirichlet characters-The nonvanishing of $L(1, \chi)$ for real nonprincipal χ .

Introduction- Dirichlet's theorem for primes of the form $4n-1$ and $4n+1$ - The plan of the proof of Dirichlet's theorem- Proof of Lemma 7.4- Proof of Lemma 7.5- Proof of Lemma 7.6- Proof of Lemma 7.7- Proof of Lemma 7.8- Distribution of primes in arithmetic progressions.

Chapters 6 & 7:- Articles 6.5 to 6.10 and 7.1 to 7.9

UNIT-II:- PERIODIC ARITHMETICAL FUNCTIONS AND GAUSS SUMS:-

Functions periodic modulo k - Existence of finite Fourier series for periodic arithmetical functions- Ramanujan's sum and generalizations- Multiplicative properties of the sums $s_k(n)$ - Gauss sums associated with Dirichlet characters-Dirichlet characters with nonvanishing Gauss sums- Induced moduli and primitive characters- Further properties of induced moduli- The conductor of a character- Primitive characters and separable Gauss sums- The finite Fourier series of the Dirichlet characters- Polya's inequality for the partial sums of primitive characters.

Chapter 8:- Articles 8.1 to 8.12.

UNIT-III:- QUADRATIC RESIDUES AND THE QUADRATIC RECIPROCITY LAW:-

Quadratic residues- Legendre's symbol and its properties- Evaluation of $(-1/p)$ and $(2/p)$ - Gauss Lemma-The quadratic reciprocity law-Applications of the reciprocity law- The Jacobi symbol- Applications to Diophantine equations- Gauss sums and the quadratic reciprocity law.

Chapter 9:- Articles 9.1 to 9.9

UNIT-IV:- PRIMITIVE ROOTS:-

The exponent of a number mod m . Primitive roots- Primitive roots and reduced residue systems- The nonexistence of primitive roots mod 2^α for $\alpha \geq 3$ - The existence of primitive roots and p for odd primes p . Primitive roots and quadratic residues- The existence of primitive roots mod p^α - The existence of primitive roots mod $2p^\alpha$ - The non existence of primitive roots in the remaining cases- The number of primitive roots mod m . The index calculus- Primitive roots and Dirichlet characters-Real-valued Dirichlet characters mod p^α -Primitive Dirichlet characters mod p^α .

Chapter- 10:- Articles 10.1 to 10.13.