

Syllabus

M 309 - Number Theory I

SX-S 338

2005-06 AB

Unit I : Arithmetical Functions and Dirichlet Multiplication

Introduction. The Möbius function $\mu(n)$, The Euler totient function $\varphi(n)$, A relation connecting φ and μ , A product formula for $\varphi(n)$, The Dirichlet product of arithmetical functions, Dirichlet inverses and Möbius Inversions formula, The Mangoldt function $\Lambda(n)$, Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function $\lambda(n)$, The divisor functions σ $\alpha(n)$, Generalised convolution.

UNIT II : Averages of Arithmetical Functions; Introduction, The big Oh notation Asymptotic equality of functions, Euler's summation formula, Some elementary asymptotic formulas, the average order of $d(n)$, the average order of divisor functions σ $\alpha(n)$, the average order of $\varphi(n)$, An application to the distribution of lattice points, visible from the origin, The average order of $\mu(n)$ and $\Lambda(n)$, The partial sums of a Dirichlet product, Applications to $\mu(n)$ and $\Lambda(n)$, Another identity for the partial sums of a Dirichlet product.

UNIT III : Some Elementary Theorems on the Distribution of Prime Numbers; Introduction, Chebyshev's functions $\Psi(x)$ and $\theta(x)$, Relations connecting $\pi(x)$ and $\theta(x)$, some equivalent forms of the prime number theorem, Inequalities of $\pi(n)$ and P_n , Shapiro's Tauberian theorem, Application of Shapiro's theorem, An asymptotic formulae for the partial sums $\sum_{p \leq x} (1/p)$.

UNIT IV : Congruences; Definition and basic properties of congruences, Residue classes and complete residue systems Linear congruences, Reduced residue systems and Euler-Fermat theorem, Polynomial congruences module p , Lagrange's theorem, Simultaneous linear congruences, the Chinese remain-

der theorem, Applications of the Chinese remainder theorem, Polynomial congruences with prime power moduli.

Content and Extent as In the Textbook :

Introduction to Analytic Number Theory - by Tom M. Apostol, Narosa Publishing House, New Delhi.

(Sections 2.2 to 2.14, 3.1 to 3.12, 4.1 to 4.9, 5.1 to 5.9.)