MA/MSc - Mathematics (Previous)
Assignments
ALGEBRA

Answer any five of the following.
All questions carrying equal marks 5x4=20 Marks

1. (a) Show that any group is isomorphic to a permutation group.

   (b) Define the normalizer $N(H)$ of H in a group G and show that $N(H)$ is the largest in which H is normal.

2. (a) State and prove the first isomorphism theorem for groups.

   (b) State and prove Burnside theorem.

3. Let G be a finitely generated abelian group. Then show that G can be decomposed as a direct sum of finite number of cyclic groups $c_1, c_2, ... c_n$ such that $G = C_1 \oplus C_2 \oplus ..... \oplus C_m$ with either all the C_i;s are infinite or for some $1 \leq r \leq m$, $C_1, C_2 ..... C_r$ are of orders $n_1, n_2 .... n_r$ such that $n_1 | n_2 .... n_{r-1} | n_r$ and $C_{r+1} ..... C_m$ are infinite.

4. (a) State and prove Sylow's first theorem.

   (b) Show that any group of order p^2, p prime is abelian.

5. (a) Let $f : R \rightarrow S$ be a homomorphism of a ring R into ring S and let $N = \ker f$. Then show that the mapping $F : A \rightarrow f(A)$ defines a 1 - 1 correspondence from the set of ideals in R that contain N onto the set of all ideals in S.

   (b) Show that in any non-zero R with unity 1, every proper ideal is contained in a maximal ideal.

6. (a) Let R be a P.I.D. with identify. Let p be a non zero proper ideal of R. Show that p is prime if and only if p is maximal.

   (b) Show that an Euclidean domain is a P.I.D.

7. (a) State and prove Eisenstien's criterion.

   (b) Find the degree of the extension $\varphi(\sqrt{2}, \sqrt{3})$ over φ.

8. Let E be an extension of the field F. Let $u \in E$ be algeraic over F. Let $p(x) \in f[x]$ be a minimal polynomial of least degree such that $p(u) = 0$. Then show that

   (a) $p(x)$ is irreducible of F.

   (b) if $g(x) \in F[x]$ such $g(u) = 0$. Then $p(x)/g(x)$.

   (c) There is exactly one monic polynomial $p(x) \in f[x]$ of least degree such that $p(u) = 0$.

MA/MSc - Mathematics (Previous)
Assignments

PAPER-II: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Answer any five of the following.
All questions carrying equal marks 5x4=20 Marks

---

1. Prove that $T \in L(V)$ is triangulable if and only if its minimal polynomial p has the form $p(x) = (x - \lambda_1)^{ri} \ldots\ldots (x - \lambda_k)^{rk}$, $U \leq ri \leq k$, $i \in \{1, \ldots, k\}$.

2. (a) Prove that for any $T \in L(R^n)$ there exist unique operations S, N and R^n such that T = S + N, SN = NS, S is semi simple N is nilpotent.

   (b) Let $Q \in L(\mathbb{C})$. Then prove that $Q = T_{\mathbb{C}}$ if and only if $Q\sigma = \sigma Q$.

3. Discuss the method of variation of parameters and find a particular solution of $y'' + y = cosec\ x$.

4. Discuss the method of solving the Bernoull's equation and hence solve $xy' + y = x^4 y^3$.

5 (a) Prove that any linear combination of two solutions of first and homogeneous system in also a solution.

   (b) Find the general solution of the system $n' = 3m + 2n;\ n' = -5m+n$.

6. Derive Volterr's prey predakor equations and discuss its dynamics.

7. (a) Solve 0^{th} order Bessel's equation using Laplace transform.

   (b) State and prove homomorphic property of convolution.

8. Discuss the methods of solving an integral equation using convolution method and hence solve

   $y(x) = x3 + \int_0^x sin(x - t)y(t)\ dt.$

MA/MSc - Mathematics (Previous)
Assignments

TOPOLOGY

Answer any five of the following.
All questions carrying equal marks 5x4=20 Marks

1. (a) Let *(X, d)* be a matrix space. Prove that a subset *G* of *X* is open if and only if *G* is a union of open spheres.

   (b) State and prove Cantor's intersection theorem.

2. (a) Let *X* and *Y* be metric spaces. Let $f : X \to Y$. Prove that *f* is continuous if and only if inverse image of every open set in Y is open in *X*.

   (b) State and prove Cauchy's inequality.

3. (a) Prove that the every second countable space is separable. Is converse true? Justify your answer.

   (b) State and prove Lindelof's theorem.

4 (a) State and prove Heine-Borel theorem.

   (b) State and prove Tychnoff's theorem.

$$\bigcap_{i\in I} A_i \neq \phi$$

5. (a) State and prove Urysohn's lemma.

   (b) Let *X* be a topological space. Let $\{A_i\}_{i\in I}$ be a nonempty family of connected subspaces of X such that . Prove that $\bigcap_{i\in I} A$ is connected.

6. (a) Prove that the components of any totally disconnected space and its points.

   (b) Prove that any Hausdorff space with a dopen base is totally disconnected.

7. (a) State and prove Weirstrass approximation theorem.

   (b) State and prove the real stone Weirstrass theorem.

8. (a) State and prove the complex stone Weirstass.

   (b) Prove that any locally compact Hausdorff space in completely regular.

MA/MSc - Mathematics (Previous)
Assignments

PAPER-V : DISCRETE MATHEMATICS

Answer any five of the following.
All questions carrying equal marks 5x4=20 Marks

1. (a) Define graph. Prove that the sum of the degrees of all vertices in a graph is even. Deduce that the number of vertices of odd degree in any graph is even.

   (b) Prove that a graph is bipartite if and only if it contains no odd cycles.

2 (a) Define tree. Prove that any tree with n vertices contain $n - 1$ edges.

   (b) Define Eulerian graph. Prove that a connected multigraph is Eulerian if and only if the degree of each of its vertices is even.

3 (a) Prove that every distributive lattice is modulas. Is converse true? Justify your answer.

   (b) Define ideal of a lattice. Prove that a non empty subset I of a Boolean algebra B is an ideal of B if and only if " $a \vee b \in I \Leftrightarrow a \in I$ and $b \in I$ ".

4. (a) State and prove Demorgan laws in a Boolean algebra.

   (b) Using Quine McCluskey algorithm find the minimal form of the Boolean polynomial.

   $f(x,y,z,u) = x'yz'u + x'y'z'u + xyz'u' + x'yzu + xyz'u' + xyz'u + xyzu' + xyzu.$

   (c) State and prove representation theorem for finite Boolean algebras.

5. (a) Define a semi automation and explain its representation as
   (i) diagram and
   (ii) tabular form.

   (b) Prove that every semigroup can be embedded in a monoid.

6. (a) Given any monoid $(S, 0)$, prove that there exists an automation whose monoid isomorphic to $(S, 0)$.

   (b) Minimize the following automata:

δ	a_1	a_2	λ	a_1	a_2
z_1	z_2	z_7	z_1	0	0
z_2	z_2	z_7	z_2	0	0
z_3	z_5	z_1	z_3	0	0
z_4	z_6	z_2	z_4	0	0
z_5	z_5	z_3	z_5	1	0
z_6	z_6	z_4	z_6	1	0
z_7	z_1	z_6	z_7	0	1

7. (a) Obtain the generating matrix of the (5, 2) linear code whose parity check matrix is H given by H = $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ and write all the code words.

(b) Define minimum distance of a linear code, prove that the minimum distance of a linear code is the least of the weights of all non zero code words.

8. (a) State and prove Gilbert-Varsham or Bound theorem.

(b) Find all proper cyclic codes of length 6.

MA/MSc - Mathematics (Previous)
Assignments

REAL ANALYSIS

Answer any five of the following.
All questions carrying equal marks 5x4=20 Marks

1. (a) Prove that the sub sequential limits of sequence $\{p_n\}$ in a metric space X form a closed subset of X.

   (b) Prove that if $p > 1$ then the series converges; and if $p \le 1$ then series diverges.

2. (a) Let f be a continuous mapping of a compact matrix space X into a metric space Y. Then prove that f is uniformly continuous on X.

   (b) Suppose f is continuous on $[a,b]$, $f'(x)$ exists at some point $x \in [a,b]$, g is defined on a interval I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t)),\ a \le t \le b$ then prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$.

3. (a) State and prove necessary and sufficient condition for the existence of Riemann-Stieltjes integral of bounded function of $[a,b]$.

   (b) Suppose f $\in$ (α) on [a,b], $m \le f \le M$, φ is continuous on $[m, M]$, and $h(x) = \varphi(f(x))$ on $[a,b]$. Then prove that $h \in$ (α) on $[a,b]$.

$$\sum_{n=2}^{k} \frac{\in\ (\alpha)}{n(\log n)^p}$$

4. (a) Suppose f is a real, continuously differentiable function on $[a,b]$, $f(a) = f(b) = 0$ and $\int_a^b f^2(x)dx = 1$. Then prove that $\int_a^b xf(x)f'(x)dx = -\frac{1}{2}$ and that $\int_a^b [f'(x)]^2 dx \,.\, \int_a^b x^2 f^2(x)dx > \frac{1}{4}$.

   (b) Suppose $\overline{f}$ maps $[a,b]$ into and if for some monotorically increasing function α on $[a,b]$ then prove that $\overline{f}$ $\in$ (α), *and* $\left\| \int_a^b \overline{f}\, d\alpha \right\| \le \left\| \int_a^b f \right\| d\alpha$.

5. (a) Suppose k is compact, and

   (i) $\{f_n\}$ is a sequence of continuous functions on K.

   (ii) $\{f_n\}$ converges point wise to a continuous function f on K.

   (iii) $f_n(x) \ge f_{n+1}(x)$ for all $x \in K$, $n = 1,2,....$ then prove that $f_n \to f$ uniformly on K.

(b) Prove that there exists a real continuous function on the real line which is now here differentiable.

6. (a) If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,...$ and if $\{f_n\}$ is converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

(b) Suppose the series and $\sum b_n x^n$ converge in the segment S = (-R,R). Let E be the set of all x $\in$ S at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$. If E has a limit point in S then prove that $a_n = b_n$ for $n = 0,1,2,...$

7. (a) Suppose $\bar{f}$ maps an open set into . Then prove that $\in \mathscr{C}'(E)$ if and only if the partial derivatives $D_j f_i$ exists and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$.

(b) State and prove contraction principle.

8. State and prove inverse function theorem.

$E \subset R^n$ R^m $\bar{f}$ $\sum a_n x^n$