



ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
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Prof. L.D. Sudhakara Bahu

DIRECTOR

Date: 10.11.2016

No. SDE/E-III/2016

TUITION FEE MEMORANDUM

Sub: M.A / M.Com / M.Sc. Maths Degree course of study through School of Distance Education payment of Tuition Fee for second year-Regarding.

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The students of Second Year M.A / M.Com / M.Sc. Maths Degree course 2015- 16 (admitted batch) are hereby informed that they have to pay the tuition fee (as noted below) towards Second year for the academic year 2015-2016 as per the following dates by way of crossed Demand Draft drawn in favour of the Registrar, Andhra University, Visakhapatnam on any nationalized bank payable at Visakhapatnam.

	M.A / M.Com./ M.A/M.Sc. Maths
Payment towards II Year Tuition Fee	Rs.2,175/-
Last Date without any penal fee	15-12-2016
With a penal fee of RS.50/- upto	16-02-2017
With a penal fee of RS.200/- on or after	17-02-2017

The Students are advised to write clearly the **purpose of the remittance and CODE NUMBER on the reverse of D.D and in the letter enclosed.** Challans, Money Orders, Postal Orders and S.B.I Challans, etc. will not be accepted.

Defaulters of Tuition fee to this School will not be supplied with the relevant reading material and their examination applications will not be processed for the ensuing examinations.

Candidates who have already paid the said tuition fee are advised to furnish the payment particulars through a letter. Lessons will be dispatched as and when the tuition fee is received. Academic calendar for the academic year 2016-2017 will be dispatched direct to the candidates in due course.

With best wishes,

Yours sincerely
L.D. SUDHAKARA BABU
DIRECTOR

Note: 1) Irrespective of the appearances at the University examinations etc.; the consequent result, the candidate is deemed to have entered into the Final year of study.

2) Details of Academic programmes will be intimated shortly.

SCHOOL OF DISTANCE EDUCATION

ANDHRA UNIVERSITY - VISAKHAPATNAM-530 003

PROF L D SUDHAKAR BABU

Director

MA/MSC MATHEMATICS

WEEK – END CLASSES AT DIFFERENT CENTERS

Sl.No	Name of the center	Date
1	Andhra University (Campus) Department of maths	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1,8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,11,12,19,26,27
2	P R Government Collage : Kakinada	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,
3	Government College: Rajhmundru	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,
4	K G R L College : Bhimavaram	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,
5	C R R College Eluru	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,
6	Government College : Srikakulam	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,
7	Maharaja College : Vijayanagaram (Phoolbhag)	DECEMBER - 2016 : 11,12,18,24,25 JANUARY - 2017 : 1, 8,22,29 FEBUARY - 2017 : 5,11,12,19,26 MARCH -2017 : 5,

All the students who intend to attend the programme may kindly report with their Identity cards at reception counter at the respective venue at 9.00 a.m. for the Programme. Time-Table and other instructions about the classes will be given at the time of Registration.

With best wishes.

PROF. P. HARI PRAKASH
Course Coordinator

Yours Sincerely,
PROF L D SUDHAKAR BABU
DIRECTOR

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

COMPLEX ANALYSIS

Answer any Five questions

All questions carry equal marks.

(5x4=20 marks)

Unit I

1. Let f and g be analytic on G and respectively and suppose $f(G) \subset \Omega$. Then with the usual notation show that $g \circ f$ is analytic on G and

$$(g \circ f)'(z) = g'(f(z))f'(z) \text{ for all } z \text{ in } G.$$

2. Prove that a Möbius transformation takes circles onto circles.
3. If γ is piecewise smooth and $f: [a, b] \rightarrow \mathbb{C}$ is continuous then show that

$$\int_a^b f d\gamma = \int_a^b f(t) \gamma'(t) dt, \text{ with the usual notation.}$$

Unit II

4. State and prove the first version of Cauchy's integral formula.
5. If f has an isolated singularity at a , then show that the point $z = a$ is a removable singularity iff

$$\lim_{z \rightarrow a} (z - a) f(z) = 0$$

6. Obtain the Laurent series development of $f(z) = \frac{1}{z(z-1)(z-2)}$ in the following annuli

(i) $(0; 0, 1)$ and

(ii) $(0; 1, 2)$

7. Show by complex integration that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

Unit III

8. Prove with the usual notation that $\mathfrak{E}(G, \Omega)$ is a complete metric space.
9. With the usual notation that if $|z| \leq 1$ and $P \geq 0$ then show that $|1 - E_p(z)| \leq |z|^{P+1}$.
10. State and prove the Weierstrass factorization theorem.

11. Prove that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$.

Unit IV

12. State and prove Mittag-Leffler's theorem.
13. Explain the notion of an analytic function along a given path.
14. Define a harmonic function with the usual notation that if u is harmonic then show that $f = u_x - iu_y$ is analytic.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION
Mathematics
MEASURE THEORY AND FUNCTIONAL ANALYSIS

Answer any Five questions
All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Prove that the outer measure of an interval is its length.
2. Let $\{E_n\}$ be a sequence of measurable sets. Then show that $m(\cup E_n) \leq \sum m(E_n)$. If the sets E_n are pairwise disjoint, then show that $m(\cup E_n) = \sum m(E_n)$.
3. State and prove bounded convergence theorem.
4. Let f be a non negative function which is integrable over a set E . Then show that for a given $\epsilon > 0$ there is a simple function f_1 such that for every set $A \subset E$ with $m(A) < \delta$ and $\int_A f < \epsilon$.

Unit II

5. State and prove monotone convergence theorem.
6. Show with the usual notation that if f is integrable with respect to m , then given $\epsilon > 0$ there is a simple function f_1 such that $\int |f - f_1| dm < \epsilon$.
7. State and prove Randon-Nikodym theorem.

Unit III

8. Define a normed linear space and a Banach space. Let M be a closed linear space of a normed linear space N . If the norm of a coset $x + M$ in the quotient space N/M is defined by :
$$\|x + M\| = \inf \{\|x + m\| : m \in M\}$$
, then show that N/M is a normed linear space. Further show that if N is a Banach space the N/M is also a Banach space.
9. State and prove Hahn-Banach theorem.
10. If N is a normed linear space and x_0 is a non-zero vector in N , then show that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

Unit IV

11. State and prove Bessel's inequality.
12. If T is an operator on H , then show that the following conditions are all equivalent to one another :
 - (i) $T^* T = I$
 - (ii) $(Tx, Ty) = (x, y)$ for all x in H with the usual notation.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

NUMBER THEORY

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. If $n \geq 1$, prove that $\sum_{d|nq} Q(d) = n$.
2. For $n \geq 1$ prove that $Q(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.
3. Show that the set of lattice points visible from origin has density $6/p^2$.

Unit II

4. Show that for any integer $n \geq 2$, we have $\frac{1}{2} \frac{n}{\log n} < p(n) < 6 \frac{n}{\log n}$.
5. Assume $(a, h) = d$. Then show that the linear congruence $ax = b \pmod{n}$ has solution if and only if $d|b$.
6. Solve the congruence $5x = 3 \pmod{24}$.

Unit III

7. For any real-valued non principal character $\chi \pmod{k}$, let $A(n) = \sum_{d|n} \chi(d)$ and

$$B(x) = \sum_{n \leq x} \frac{A(n)}{\sqrt{n}}.$$

Then show that

$$B(x) \rightarrow \infty \text{ as } x \rightarrow \infty.$$

8. $B(x) = 2\sqrt{x}L(1, \chi) + o(1)$ for all $x \geq 1$. There for $L(1, \chi) \neq 0$.
9. Show that there are infinitely many primes of the form $4k + 1$.
10. For $x > 1$ and $x \neq x_1$, show that $L(1, \chi) \sum_{n \leq x} \frac{m(n) \chi(m)}{n} = o(1)$

Unit IV

11. Let P be an odd prime. Then show that for all n we have $(n/P) = M^{\frac{(P-1)}{2}} \pmod{r}$.
12. For every odd prime, show that $(2/P) = (-1)^{\frac{(P^2-1)}{8}} = \begin{cases} 1 & \text{if } P \equiv \pm 1 \pmod{8} \\ -1 & \text{if } P \equiv \pm 3 \pmod{8} \end{cases}$.
13. Determine those odd primes P for which 3 is a quadratic residue and those for which it is a non residue.
14. State and prove reciprocity law for Jacobi symbols.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

(Optional) PAPER II -- LATTICE THEORY

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Define a semi lattice. If (S, \cdot) is a semi lattice, prove that there exists a partial ordering \leq on S Such that for any x, y in S , $\text{lub}\{x, y\} = x.y$.
2. Let L and M be non empty sets. Let $f : L \rightarrow M$ be a bijection. If (L, \leq) is a lattice, prove that there exists a partial ordering \leq on M such that (M, \leq) is also a lattice and $f : L \rightarrow M$ is a lattice isomorphism.
3. Let (L, \leq) be a lattice. Prove that a non empty subset I of L is an ideal of L if and only if " $a \in I, b \in I \Leftrightarrow a \vee b \in I$ ".
4. Define (i) weakly complemented lattice (ii) Semi complemented lattice. Prove that any weakly complemented lattice is semi complimented.

Unit II

5. State and prove fix element theorem for complete lattices.
6. Prove that every element of a lattice satisfying maximum condition is compact.
7. Prove that a modular lattice is distributive if no sublattice of it is isomorphic to M_3 .
8. Is the converse of above result is true ? Justify.

Unit III

9. Prove that the set of all complemented elements in a bounded distributive lattice is a Boolean algebra.
10. Prove that any complete Boolean algebra satisfies infinite meet distributive law.
11. Let $(R, +, \cdot)$ be a Boolean ring with identity 1. Define the binary operations \wedge, \vee and the unary operation $'$ on R by
$$a \wedge b = a.b$$
$$a \vee b = a + b + a.b$$
$$a' = 1 + a.$$
Prove that $(R, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.
12. Define valuation of a Boolean algebra and prove that a valuation f of a Boolean algebra is additive if and only if $f(0) = 0$.

Unit IV

13. If L is a distributive lattice, prove that the set of all ideals of L is a distributive lattice under set inclusion.
14. Let $\theta_1, \theta_2, \theta_3$ be congruence relations on a set A such that θ_1 and θ_3 are permutable and $\theta_1 \leq \theta_3$, prove that $\theta_1 \leq (\theta_2 \wedge \theta_3) = (\theta_1 \wedge \theta_2) \wedge \theta_3$.
15. Prove that a lattice L is distributive if and only if L is isomorphic to a ring of sets.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION
Mathematics
(Optional) LINEAR PROGRAMMING AND GAME THEORY

Answer any Five questions
 All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. State the general linear programming problem (LLP) and write it in the standard form. Discuss the different types of probable solution that may exist for a LPP.
2. Explain the principle of duality in LPP. Relate the concept of a shadow price to the dual of a LLP.
3. State and prove the optimality criterion for the standard maximum LPP.
4. Prove that the set of all feasible solutions of a LPP form a convex set.

Unit II

5. Prove that a necessary and sufficient condition for any LPP and its dual to have an optimum solution is that both have feasible solutions.
6. Solve the following canonical maximum problem by
 $4x_1 + 2x_2 + x_3 = 4, x_1 + 3x_2 = 5, 2x_1 + 3x_2 = \text{Maximum}$.
7. In the course of simplex table computation of solving a LPP, describe how you will detect a degenerate, an unbounded and infeasible solution? How do you resolve degeneracy if it arises in a particular case?
8. Solve the following LPP using simplex method

Maximize $Z = 3x_1 + 2x_2$

Subject to

$$2x_1 + x_2 \leq 40,$$

$$x_1 + x_2 \leq 24,$$

$$2x_1 + 3x_2 \leq 60 \text{ and}$$

$$x_1, x_2 \geq 0$$

Unit III

9. Solve the following transshipment problem to find the optimum transportation route

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	S_1	0	2	2	1	8
	S_2	1	0	2	3	3
Destination	D_1	2	2	0	0	-
	D_2	1	3	2	0	-
Demand		-	-	7	4	

10. State and Prove the Max flow - Min cut theorem.
11. Solve the following assignment problem

$$\begin{array}{c}
 J_1 \quad J_2 \quad J_3 \quad J_4 \\
 A \left(\begin{array}{cccc} 10 & 12 & 19 & 11 \\ 5 & 10 & 7 & 8 \\ 12 & 14 & 13 & 11 \end{array} \right) \\
 B \\
 C
 \end{array}$$

12. Solve the following transportation problem to find an optimum solution.

		Markets				Supply
		M ₁	M ₂	M ₃	M ₄	
Plants	P ₁	11	13	17	14	250
	P ₂	16	18	14	10	300
	P ₃	21	24	13	10	400
Demand		200	225	275	250	

Unit IV

13. Establish and explain the relation between matrix games and linear programming.
 14. Solve the following game by linear programming method

Player B

$$\text{Player A} \begin{pmatrix} -1 & 1 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & -3 \end{pmatrix}$$

15. Explain saddle point and principle of dominance as applied to a game. Explain the Min-Max principle.
 16. Solve the following game using the principle of dominance

Player B

$$\text{Player A} \begin{pmatrix} 2 & -2 & 4 & 1 \\ 6 & 1 & 12 & 3 \\ -3 & 2 & 0 & 6 \\ 2 & -3 & 7 & 7 \end{pmatrix}$$

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

UNIVERSAL ALGEBRA

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Prove that every chain is a modular lattice.
2. Prove that every distributive lattice is modular.
3. Define the terms (i) Semilattice (ii) Boolean algebra and give an example of each.
4. Let A and B be some type of algebras and $\alpha : A \rightarrow B$ be mapping. If α is an embedding then prove that $\alpha(A)$ is a subuniverse of B.
5. Define the terms (i) factor congruence (ii) directly indecomposable algebra.
6. In a Boolean algebra B, prove that $(a \vee b)' = a' \wedge b'$.
7. Prove that every Boolean ring is commutative and is of characteristic 2.
8. Let F be a filter of a Boolean algebra B, then prove that $F' = \{a' \mid a \in F\}$ is an ideal of B.

Unit I

9. Prove that L is non modular lattice iff \mathbb{N}_5 can be embedded into L.
6. Prove that if L is a distributive lattice then the set of ideas I(L) and L form a distributive lattice.
7. Let P be a peset such that $\forall A$ exists for every subset A. Then prove that P is a complete lattice.
8. Prove that every algebraic lattice is isomorphic to the lattice of closed subsets of some set A with an algebraic closure operator C.

Unit II

9. State and prove the Irredundant Basis Theorem.
10. Prove that for A an algebra there is an algebraic closure operator q on $A \times A$ such that the closed subsets of $A \times A$ are precisely the congruences on A . Hence con A is an algebraic lattice.
11. Prove that if A is congruence-permutable, then A is congruence modular.

Unit III

13. Prove that an algebra A is directly idecomposable iff the only factor congruences on A are Δ and ∇ .
14. Prove that every algebra A is isomorphic to a subdirect product of subdirectly irreducible algebras.
15. Let K be a finite set of finite algebras, then prove that $V(K)$ is locally finite variety.
16. Prove that if V is a variety and X is a an infinite set of variables then $\mathcal{V} = M(\text{Idv}(X))$.
17. Prove that every finite Boolean algebra is isomorphic to the Boolean algebra of all subsets of some finite set X.
18. Let $(B, \vee, \wedge, ', 0, 1)$ be a Boolean algebra. Define operations + and . on B by
$$a + b = (a \vee b') \vee (a' \wedge b)$$
 and $ab = a \wedge b$.
Then prove that $(B, +, \cdot, ', 0, 1)$ is a Boolean ring.
19. Let B be a Boolean algebra. If q is a binary relation on B, then prove that q is congruence on B iff $0/q$ is an ideal, and for $a, b \in B$, we have $(a, b) \in q$ iff $a + b \in 0/q$.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

(Optional Paper) : INTERGAL EQUATIONS

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Form the intergal equation corresponding to the differential equation $y'' + 2xy' + y = 0$, $y(0) = 1, y'(0) = 0$.
2. Show that the integral equation $xf(x) = I \int_0^x \exp\{(x-y)\}f(y) dy; x \geq 0$ possess a continuous unbounded solution for $x \geq 0$.
3. Prove that a Volterra integral equation of the first kind can be reduced to a Volterra integral equation of the second kind.
4. Let A and B be some type of algebras and $\alpha : A \rightarrow B$ be mapping. If α is an embedding then prove that $\alpha(A)$ is a subuniverse of B.
5. Define the terms (i) factor congruence (ii) directly indecomposable algebra.
6. In a Boolean algebra B, prove that $(a \vee b)' = a' \wedge b'$.
7. Prove that every Boolean ring is commutative and is of characteristic 2.
8. Let F be a filter of a Boolean algebra B, then prove that $F' = \{a' \mid a \in F\}$ is an ideal of B.

Unit I

9. Prove that L is non modular lattice iff N_5 can be embedded into L.
6. Prove that if L is a distributive lattice then the set of ideas I(L) and L form a distributive lattice.
7. Let P be a peset such that $\forall A$ exists for every subset A. Then prove that P is a complete lattice.
8. Prove that every algebraic lattice is isomorphic to the lattice of closed subsets of some set A with an algebraic closure operator C.

Unit II

9. State and prove the Irredundant Basis Theorem.
10. Prove that for A an algebra there is an algebraic closure operator q on $A \times A$ such that the closed subsets of $A \times A$ are precisely the congruences on A . Hence con A is an algebraic lattice.
11. Prove that if A is congruence-permutable, then A is congruence modular.

Unit III

13. Prove that an algebra A is directly idecomposable iff the only factor congruences on A are Δ and ∇ .

14. Prove that every algebra A is isomorphic to a subdirect product of subdirectly irreducible algebras.
15. Let K be a finite set of finite algebras, then prove that $V(K)$ is locally finite variety.
16. Prove that if V is a variety and X is a an infinite set of variables then $V = M(Ids(X))$.
17. Prove that every finite Boolean algebra is isomorphic to the Boolean algebra of all subsets of some finite set X .
18. Let $(B, \vee, \wedge, ', 0, 1)$ be a Boolean algebra. Define operations $+$ and \cdot on B by

$$a + b = (a \vee b') \vee (a' \wedge b) \text{ and } a \cdot b = a \wedge b.$$
 Then prove that $(B, +, \cdot, ', 0, 1)$ is a Boolean ring.
19. Let B be a Boolean algebra. If \mathbf{q} is a binary relation on B , then prove that \mathbf{q} is congruence on B iff $0/\mathbf{q}$ is an ideal, and for $a, b \in B$, we have $(a, b) \in \mathbf{q}$ iff $a + b \in 0/\mathbf{q}$.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

COMMUTATIVE ALGEBRA

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Let P_1, P_2, \dots, P_n be prime ideals and let I be an ideal of a ring A such that $I \subseteq \bigcup_{i=1}^n P_i$. Then prove that $I \subseteq P_i$ for some i .
2. Let I_1, I_2, \dots, I_n be ideals and P be prime ideal containing $\bigcup_{i=1}^n I_i$. Then prove that $P \supseteq I_i$ for some i . Further if $P \subseteq \bigcap_{i=1}^n I_i$ then prove that $P = I_i$ for some i .
3. Let M be a finitely generated A -module. Let I be an ideal of A . Let f be an endomorphism of M such that $f(M) \subseteq IM$. Then prove that f satisfies an equation of the form $f^n + a_1 f^{n-1} + \dots + a_n = 0$ where a_1, a_2, \dots, a_n are in I .
4. Let $M' \xrightarrow{u} M \rightarrow M'' \rightarrow O$ be a sequence of A -modules and homomorphism. Then prove that (i) is exact if and only if for all A -modules N , the sequence $O \rightarrow \text{Hom}(M'', N) \xrightarrow{\bar{u}} \text{Hom}(M, N) \xrightarrow{\bar{u}} \text{Hom}(M', N)$ (ii) is exact.

Unit II

5. For an A module N prove that the following are equivalent.
 - (a) N is flat
 - (b) If $O \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow O$ is any exact sequence of A -modules then the tensored sequence $O \rightarrow M' \otimes N \rightarrow M \otimes N \rightarrow M'' \otimes N \rightarrow O$ is exact.
 - (c) If $f : M' \rightarrow M$ is injective then $f \otimes I : M' \otimes N \rightarrow M \otimes N$ is complete
 - (d) If $f : M' \rightarrow M$ is injective, and M, M' are finitely generated then $f \otimes I : M' \otimes N \rightarrow M \otimes N$ is injective.
6. Describe the construction of the ring of fractions of a ring A with respect to multiplicatively closed subset S of A .
7. Let M, N, P be A -module and S be a multiplicative set in A and let $f : M \rightarrow N$ and $g : N \rightarrow P$ be A -module homomorphism. Then prove that $S^{-1}(g \circ f) = (S^{-1}g) \circ (S^{-1}f)$.
8. State and prove First Uniqueness theorem.
9. Let A be a subring of a ring B and C be the integral closure of A in B . Let S be a multiplicative set in A . Then prove that $S^{-1}C$ is the integral closure of $S^{-1}A$ in $S^{-1}B$.

Unit IV

10. Let $O \rightarrow M' \xrightarrow{f} M \xrightarrow{g} M''$ be an exact sequence of A -modules. Prove that M is Noetherian if and only if M' and M'' are Noetherian.
11. State and prove Hilbert Basis theorem.
12. Prove that in an artin local ring, every element is either a unit or nilpotent.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION
Mathematics
NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES

Answer any Five questions
All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Write FORTRAN expressions corresponding to

(i) $\frac{1 - e^{-a\sqrt{x}}}{1 + xe^{-|x|}}$

(ii) $\sqrt{x - y^2} - \frac{z^3}{\sin(a + b)}$

(iii) $ae^{-kt} + \frac{bx + c}{cx - d}$

2. Find the final value of A in the following program A = 2.56, A = (A + 0.05)*10, I - A, A = I, A = A/10.

3. Write a FORTRAN program to read the radius of a circle in cms and compute its circumference and area of it.

4. Write short notes on the following :

- (i) Generalised input -output statements
- (ii) Dimension statement
- (iii) Arithmetic expressions.

5. Write a FORTRAN program to find the product of two matrices which are conformable for multiplication.

Unit II

6. Given the following data of x and $f(x)$, find $f'(6, 0)$ and $f''(6, 2)$ using the methods based on linear and quadratic interpolation. Obtain the bound (upper) on the error.

$x:$	6.0	6.1	6.2
$f(x)$	0.1750	-0.1998	-0.2223

7. Evaluate $\int_0^p \sin x dx$ using composite Simpson's $\frac{1}{3}$ rule, taking six sub intervals of equal length.

8. Evaluate $\int_{-1}^1 (1 - x^2)^{3/2} \cos x dx$ using Gauss Legendre 3-point formula.

9. Evaluate $I = \int_0^1 \frac{dx}{1 + x^2}$ using Trapezoidal rule with $h = 0.5, 0.25, 0.125$ successively. Improve your result with Romberg method.

Unit III

10. Given that $\frac{dy}{dx} = y - x$; $y(0) = 2$ compute $y(0,2)$ using Runge-kutta method of order four by taking $h = 0.1$.

11. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using predictor-corrector method with $P : u_{j+1} = u_j + \frac{h}{2}(3u'_j - u'_{j-1})$
- $C : u_{j+1} = u_j + \frac{h}{2}(u'_{j+1} - u'_j)$ as PM_pCM_c .
12. Using Taylor series method of order two, compute $y(0.2)$, given that $\frac{d^2y}{dx^2} = y^2 + x$, $y(0) = 1$.
Take $h = 0.1$.
13. Solve the boundary value problem $u'' = u + x$, $x \in [0, 1]$, $u(0) = 0$, $u(1) = 0$ using the shooting method. Use Euler's method to solve the initial value problem with $h = 0.2$.
14. Write a FORTRAN program to evaluate $\int_1^5 \frac{dx}{x}$ using composite trapezoidal rule with $h = 1$.
15. Write a FORTRAN program to obtain a numerical solution to the initial value problem $u' = -2tu^2$, $u(0) = 1$ on $[0, 1]$ using Euler's method with $h = 0.2$.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

COMMUTATIVE ALGEBRA

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. If $\sum a_n$ converges absolutely then $\sum a_n$ converges

The geometric series $\sum z^n$ converges if $|z| < 1$ and diverges if $|z| \geq 1$.

2. Any mobius transformation takes circles onto circles. State and prove orientation principle.

Unit II

3. Let y be a piecewise smooth function and f is a continuous function on $[a,b]$ then

$$\int_a^b f dy = \int_a^b f(t) y'(t) dt.$$

State and prove Leibniz's rule.

4. If $\gamma : [0,1] \rightarrow C$ is a closed rectifiable curve and a not in $\{\gamma\}$ then $\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$ is an integer. Write the Laurent series expansion of $f(z) = 1/(z-1)(z-2)$.

Unit III

5. $(C(G, \Omega), r)$ is a complete metric space.

6. Let $\{f_n\}$ be a sequence in $H(G)$ such that $f_n \in C(G, C)$ such that $f_n \rightarrow f$ then f is analytic and $f_n^k \rightarrow f^k, \forall k \geq 1$.

7. A family $\mathfrak{F} \subseteq H(G)$ is normal if and only if \mathfrak{F} is locally bounded. If $|z| \leq 1$ and $p \geq 0$ then

$$|1 - E_p(z)| \leq |z|^{p+1}.$$

Unit IV

8. State and prove Runge's theorem.

9. State and prove Schwarz Reflection Principle.

10. If $m: G \rightarrow R$ is a harmonic function and $B(a, r) \subseteq G$. If γ is the circle $|z-a|=r$ then

$$(a) = \frac{1}{2\pi} \int_0^{2\pi} (a + re^{iq}) dq.$$

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

ALGEBRA

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. If H and K are subgroups whose orders are relatively prime, prove that $H \cap K = \{e\}$.
2. If H and K are subgroups of order P and n respectively, where P is prime, then either $H \cap K = \{e\}$ or $H < K$.
3. State and prove first isomorphism theorem for groups.
4. Let G be a group, and let G' be the derive group of G prove the following.
 - (i) $G' > G$
 - (ii) G/G' is abelian
 - (iii) If $H > G$, then G/H is abelian if and only if $G' \subset H$.
5. Define the action of a group on a set. Let G be a group and X be a non empty set. Prove the following :
 - (i) If X is a G-set, prove that the action of G on X induces a homomorphism of G into S_x .
 - (ii) Every homomorphism of G into S_x induces an action of G on X.

Unit II

6. Prove that every finite group is isomorphic to a subgroup of the alternating group A_n for some $n > 1$.
7. Let G be a finite abelian group of order m, prove that there exists a unique sequence m_1, m_2, \dots, m_n of positive integers such that
 - (i) $1 < m_1 | m_2 | \dots | m_n$
 - (ii) $m = m_1 m_2 \dots m_n$ and
 - (iii) $G = C_1 \oplus C_2 \oplus \dots \oplus C_n$, where each C_i is a cyclic group of order m_i ($1 \leq i \leq n$).
8. State and prove first Sylow theorem. Deduce Cauch's theorem.
9. Show that a group of order $p^r q$, p and Q distinct primes, must contain a normal Sylow sub-group and be solvable.

Unit III

10. Let R be a principal ideal domain with identity. Let P be a non zero proper ideal of R. Prove that P is prime if and only if P is maximal.
11. Let R be a commutative ring with unity in which each ideal is prime. Prove that R is a field.
12. Let R be a Boolean ring. Prove that each prime ideal $P \neq R$ is maximal.

13. Define the concept of a Unique factorisation domain. Prove that every element of a unique factorisation domain R can be expressed as a finite product of irreducible factors which is unique to within order and unit factors.
14. Let $F[x]$ be polynomial ring over a field F . Show that a non zero polynomial $f(x) \in F[x]$ is a unit iff $f(x) \in F$.

Unit IV

15. Define algebraic extension and prove that any finite extension of a field F is an algebraic extension of F .
16. Prove that $\mathcal{Q}(\sqrt{2}, \sqrt{3}) = \mathcal{Q}(\sqrt{2} + \sqrt{3})$.
17. State and prove Eisenstein's criterion and deduce that the polynomial $x^4 - 3x^2 + 9$ is irreducible over \mathcal{Q} .
18. Prove that if E is finite extension of F , then E is an algebraic extension of F .

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION
Mathematics
PAPER II - LINEAR ALGEBRA, DIFFERENTIAL EQUATIONS AND MODELS

Answer any Five questions
All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n .
2. Let T be a linear operator on an n-dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.
3. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form.

$$p(x) = (x - c_1)(x - c_2) \dots (x - c_k)$$

where c_1, c_2, \dots, c_k are distinct elements of F.

4. If $T \in L(R^4)$ has the matrix $A = \begin{bmatrix} 0 & 0 & 0 & -8 \\ 1 & 0 & 0 & 16 \\ 0 & 1 & 0 & -14 \\ 0 & 0 & 1 & 6 \end{bmatrix}$ with respect to the standard base, then

find the real canonical form of T.

Unit II

5. If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + p(x)y' + Q(x)y = 0$ on $[a, b]$ then prove that their Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on $[a, b]$.
6. Verify that $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$ and find the general solution.
7. (a) Find the general solution of

$$y'' - 2y' + 5y = 25x^2 + 12$$

by using the method of undetermined coefficients.

8. Find a particular solution of

$$y'' + 2y' + y = e^{-x} \log x$$

by using method of variation of parameters.

Unit III

9. Find the general solution of the system

$$\frac{dx}{dy} = 3x - 4y \quad \text{and} \quad \frac{dy}{dt} = x - y.$$

10. Formulate Volterra's model for the dynamics of Prey-Predator populations and find equilibrium populations of this model.
11. Solve the initial value problem

$$y' = x + y, y(0) = 1.$$

first by elementary method and then show that the Picard successive approximations of this initial value problem converge to its solution.

12. Let $f(x, y)$ be a continuous function that satisfies Lipschitz condition

$$|f(x, y_1) - f(x, y_2)| \leq k |y_1 - y_2|$$

on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem

$$y' = f(x, y), y(x_0) = y_0$$

has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$.

Unit IV

13. Find the solution of $y'' + 4y = 4x$ that satisfies the initial conditions $y(0) = 5$ by using Laplace transforms.

14. Solve the Bessel's equation of order zero

$$xy'' + y' + xy = 0$$

by using Laplace transforms.

15. Show that $L[x \cos ax] = \frac{p^2 - a^2}{(p^2 - a^2)^2}$ and use this result to find $L^{-1} = \left[\frac{1}{(p^2 - a^2)^2} \right]$.

16. Solve the integral equation

$$y(x) = x^2 + \int_0^x \sin(x-t) y(t) dt \text{ by using Laplace transforms.}$$

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

REAL ANALYSIS

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Prove that compact subset of a metric space is closed.
2. Let $\{s_n\}$ be a sequence in a metric space X and let F be set of all sub sequential limits of $\{s_n\}$ in X. Then show that F is closed in X.
3. Prove that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n=e}$.
4. State and prove Ratio test.

Unit II

5. State and prove a necessary and sufficient condition for a real function f on [a,b] to be Riemann Stieljer integrable with respect to a monotone function a on [a,b].
6. If $f \in R(a)$ then show that $|f| \in R(a)$ and $\left| \int_a^b f da \right| \leq \int_a^b |f| da$.
7. State and prove fundamental theorem of integral calculus.
8. Demonstrate integration by parts in RS integration.

Unit III

9. Prove that the limit of uniformly convergent sequence of integrable functions is integrable.
10. Establish existence of a continuous function which is nowhere differentiable.
11. State and prove the Weierstrass Approximation theorem.

Unit IV

12. Let r be a positive integer. If X is a vector space spanned by a set consisting of r vectors, then show that the number of independent vectors in X cannot exceed r.
13. Let X be a vector space. Let $A \in L(X)$. Then show that, for any two ordered bases B_1 and B_2 of X, A is invertible iff $[A: B_1, B_2]$ is invertible.
14. Let $A \in L(\mathbb{R}^n, \mathbb{R}^m)$. Then show that $|Ax| \leq K|x|$ for all $x \in \mathbb{R}^n$ iff $\|A\| \leq K$
15. $|Ax| \leq \|A\| |x|$ for all $x \in \mathbb{R}^n$.

M.A./M.Sc. (Final) (SDE) DEGREE EXAMINATION

Mathematics

PAPER V - DISCRETE MATHEMATICS

Answer any Five questions

All questions carry equal marks.

(5x4 =20 marks)

Unit I

1. Prove that a graph G is a tree if and only if any two vertices of G are connected by a unique path.
2. Prove that every connected graph contains a spanning tree.
3. Define (i) Eulerian graph and (ii) Hamiltonian graph.
4. Give an example of a Eulerian graph which is not Hamiltonian
5. Give an example of a Hamiltonian graph which is not Eulerian.

Unit II

6. Define modular lattice, Prove that the set of all normal subgroup of a group is a modular lattice under set inclusion.
7. Obtain the conjunctive and disjunctive normal forms of the Boolean polynomial $f(x_1, x_2, x_3, x_4) = (x_1 \vee x_4)$.
8. State and prove the representation theorem for finite Boolean algebras.
9. Express each of the Boolean operations \vee, \wedge and $'$ in terms of NAND operations.

Unit III

10. Define the following terms and give one example for each.
 - (i) Automata
 - (ii) Semi automata
 - (iii) Moore machine.
11. Minimize the following automaton

States	S		s	
	a1	a2	a1	a2
s0	s0	s1	0	1
s1	s0	s2	0	1
s2	s0	s3	0	1
s3	s1	s2	0	1
s4	s1	s4	0	1
s5	s5	s1	1	0
s6	s5	s1	1	0

12. Prove that every semi group can be embedded in a Monoid.
13. If F is a free semi group with a basis B, prove that F is generated by B.

Unit IV

14. Let d be the distance of a linear code Z . Prove that $d = \min\{d(u, v) \mid u, v \in Z, u \neq v\}$.
15. Prove that a linear code Z with distance d detects $d-1$ or fewer errors and there is a word of weight d which Z cannot detect.
16. State and prove Hamming Bound Theorem.
17. Let Z be a linear code over F_2 of block length $n=9$ and minimum distance 5. Find the maximum possible dimension of Z and the maximum possible value of $|Z|$.