ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER 2018-2019
M.A. / M.Sc (Previous) Mathematics
ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a homomorphism $\phi : G \to H$ is injective if and only if $ker\ \phi = \{e\}$.

   (b) Let G be a group and let G' be the derived group of G. Then prove that

   (i) $G' \lhd G$

   (ii) G/G' is abelian

   (iii) $H \lhd G$, then G/H is abelian if and only if $G' \subset H$.

2. (a) Prove that alternating group A_n is simple if $n > 4$. Consequently S_n is not solvable if $n > 4$.

   (b) State and prove Cauchy's theorem for abelian group.

3. (a) If R is a ring with unity, then prove that every ideal I in the matrix ring R_n is of the form A_n, where A is some ideal in R.

   (b) Prove that every euclidean domain is a PID.

4. (a) Let $f(x) \in \mathbb{Z}[x]$ be primitive. Then prove that $f(x)$ is reducible over Q if and only if $f(x)$ is reducible over $\mathbb{Z}$.

   (b) State and prove Eisenstein criterion.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

   (a) Let G be a group such that $a^2 = e$ for all $a \in G$. Then show that G is an abelian group.

   (b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$ as a product of disjoint cycles.

   (c) Prove that the centre of a ring is a subring.

   (d) Prove that $x^2 - 2$ is irreducible over Q.

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REAL ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) If $\{p_n\}$ is a sequence in a compact metric space X, then prove that some subsequence of $\{p_n\}$ converges to a point of X.

   (b) Let f be defined on $[a,b]$; if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then prove that $f'(x) = 0$.

2. (a) Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case, $\int_a^b f \, d\alpha = \int_a^b f(x)\alpha'(x)dx.$.

   (b) State and prove the fundamental theorem of calculus.

3. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,\mathrm{b}]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,\mathrm{b}]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,\mathrm{b}]$, to a function f *and* $f'(x) = \lim_{n\to\infty} f_n'(x) (a \le x \le b)$.

.

   (b) Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n, \; (-1 < x < 1)$. Then prove that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

4. (a) Suppose $\overline{f}$ maps on open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$ *and* $\overline{f}$ is differentiable at a point $\overline{x} \in E$. Then prove that the partial derivatives $(D_j f_i)(\overline{x})$ exist and

$$f'(\bar{x})e_j = \sum_{i=1}^{m} (D_j\ f_i)(\bar{x})\, y_i\ (1 \le j \le n).$$

(b) State and prove contraction principle.

Section - B **(4x1=4)**

5. Answer all the following.

(a) If $0 \le x < 1,$ then prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

(b) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$ then prove that $f+g \in R(\alpha)$ on $[a,b]$ and $\int_a^b (f+g)\, d\alpha = \int_a^b f\ d\alpha + \int_a^b g\, d\alpha$.

(c) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(d) If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & for\ (x,y) \ne (0,0) \\ 0 & for\ (x,y) = (0,0) \end{cases}$ then prove that $(D_1\ f)(x,y)$ and $(D_2\ f)(x,y)$ exist at every point of $\mathbb{R}^2$, although f is not continuous at $(0,0)$.

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TOPOLOGY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let X be a metric space then prove that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

   (b) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

2. (a) State and prrove Lindelof 's theorem.

   (b) Prove that every closed and bounded subspace of the real line is compact.

3. (a) Prove that every compact Hansdorff space is normal.

   (b) Let X be a Hansdorff space. If X has an open base whose sets are closed, then prove that X is totally disconnected.

4. (a) State and prove real Stone - Weirstrass theorem.

   (b) Show that $C_o(X,\mathbb{R})$ and $C_o(X,\mathbb{C})$ are closed sub spaces of $C(X,\mathbb{R})$and $C(X,\mathbb{C})$ respectively.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let X be a metric space. Prove that the metric $d : X \times X \to \mathbb{R}$ is continuous.

(b) Show that continuous image of a compact space is compact.

(c) Prove that the components of a totally disconnected space are its points.

(d) Prove that $C[a, b]$ is separable.

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M.A. / M.Sc (Previous) Mathematics
DISCRETE MATHEMATICS
Answer ALL Questions
All questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

   (b) Let $G(V,E)$ be a graph with no isolated vertex. Then show that G has an Euler circuit if and only if G is connected and the degree of every vertex of G is even.

2. (a) When do we say that a lattice is distributive. Show that a lattice L is distributive if and only if for all $a, b, c \in L$ $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

   (b) Let B be a Boolean algebra. Prove that an ideal M in B is maximal if and only if for any $b \in B$ either $b \in M$ or $b' \in M$, but not both hold.

3. (a) Define an ideal I of a semi group $(S, \odot)$. Show that a relation $\sim$ on S defined by $x \sim y :\Leftrightarrow x = y \ or \ x \in I$ and $y \in I$ is a congruence relation with respect to I. Describe $(S/\sim, \odot)$

   (b) Explain by means of an example the concept of an automation associated with a monoid $(S, \bullet)$. Show that there exists an automation whose monoid is isomorphic to $(S, \bullet)$.

4. (a) State and prove the Hamming bound theorem.

   (b) Let C be an ideal $\neq \{0\} \ of \ V_n$. Then prove that there exists a unique $g \in V_n$ with the following properties.

   (i) $g \mid x^n - 1 \ in \ F_q[x]$ (ii) $C = (g)$ (iii) g is monic.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Define a partial ordering on a set. Give an example of relation which is both a partial ordering relation and an equivalence relation.

(b) Determine the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1 x_3 + x_1' x_2)(x_2' + x_3)$$

(c) Define the group kernel of a monoid (S, o). Show that the group kernel G_s is a group within (S, o).

(d) Show that a linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.

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LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let a,b and c be elements of a field $F,$ and let A be the 3 x 3 matrix over $F: A=\begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}.$ Prove that the characteristic and minimal polynomial for A is $x^3-ax^2-bx-c\cdot$

   (b) State and prove Cayley - Hamilton theorem.

2. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y''+p(x)y'+Q(x)y=0$ on $[a,b],$ then prove that their Wronskian $W=W(y_1,\mathrm{y}_2)$ is either identically zero or never zero on $[a,b].$

   (b) Show that $y=c_1x+c_2x^2$ is the general solution of $x^2\,\mathrm{y}''-2xy'+2y=0$ on any interval not containing $0,$ and find the particular solution for which $y(1)=3$ and $y'(1)=5$.

3. (a) If the two solutions $x=x_1(t),\ y=y_1(t)$ and $x=x_2(t),\ \mathrm{y}=\mathrm{y}_2(t)$ of the homogeneous system $\dfrac{dx}{dt}=a_1(t)x+b_1(t)y;\ \dfrac{dy}{dt}=a_2(t)x+b_2(t)y$ are linearly independent on $[a,\mathrm{b}]$ and if $x=x_p(t),\ y=y_p(t)$ is any particular solution of

$$\frac{dx}{dt}=a_1(t)x+b_1(t)y+f_1(t);\frac{dy}{dt}=a_2(t)x+b_2(t)y+f_2(t) \qquad (1)$$

   on this interval, then prove that $x=c_1x_1(t)+c_2x_2(t)+x_p(t);$ $y=c_1y_1(t)+c_2y_2(t)+y_p(t)$ is the general solution of system (1) *on* $[a,b].$

   (b) Find the general solution of the system $\dfrac{dx}{dt}=-3x+4y;\ \dfrac{dy}{dt}=-2x+3y$.

4. (a) By the method Laplace transforms, find the solution of

$$y''-4y'+4y=0,\ y(0)=0\ and\ y'(0)=3.$$

(b) State and prove convolution theorem on Laplace transforms.

Section - B **(4 x 1 = 4)**

5. Answer all the following

(a) Let $T \in L(R), F = \mathbb{R}$ and matrix of T w.r.t. the standard basis is $\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$. Find the characteristic and minimal polynomials of T.

(b) Show that the functions $f(x) = x^3; g(x) = x^2|x|$ are not linearly dependent.

(c) Find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = y \end{cases}$$

(d) Find the Laplace trasform of $sin\ ax$.

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COMPLEX ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) If $\gamma : [a, b] \to c$ is piecewise smooth curve then prove that γ is of bounded variation and $V\ \gamma = \int_a^b |\gamma'(t)|\ dt$.

   (b) Let Z_1, Z_2, Z_3, Z_4 be four distinct points in $\mathbb{C}_\infty$. Then prove that (Z_1, Z_2, Z_3, Z_4) is a real number iff all four points lie on a circle.

2. (a) State and prove Phragmen - Lindel of theorem.

   (b) State and prove Residue theorem.

3. (a) State and prove Montel's theorem

   (b) State and prove Arzela - Ascoli theorem.

4. (a) State and prove Mittag - Leffler's theorem.

   (b) Let $D = \{z / |z| < 1\}$ and suppose $f : \partial D \to R$ is continuous function. Show that there exists a continuous function $\overline{u} : \overline{D} \to R$ such that

   (i) $U(z) = f(z)$ and

   (ii) $X\ \ f$ is harmonic in D.

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

(a) Define analytic function and Mobius transformation.

(b) Let $u(x, y) = e^x \cos\ y$. Show that there exists an analytic function f such that $Ref = u$.

(c) If $|z| \leq 1$ and $\rho \geq 0$ then show that $|1 - E_p(z)| \leq |z|^{P+1}$.

(d) Find the value of $\int_0^\mu \frac{d\theta}{5 + 2\cos\ \theta}$

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MEASURE THEORY AND FUNCTIONAL ANALYSIS

Answer any ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Show that the outer measure of an interval is its length.

   (b) State and prove Bounded Convergence theorem.

2. State and prove Radon - Nikodym theorem.

3. State and prove the Hahn - Banach theorem.

4. (a) If M is a closed linear subspace of Hilbert space H then show that $H = M \oplus M^{\perp}$.

   (b) Show that an operator T on H unitary $\Leftrightarrow$ it is an isometric isomorphism of H onto itself.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Prove that $m*$ is translation invariant.

(b) Let (X, B, μ) be a finite measure space and g an integrable function such that for some constant M, $\left|\int g\varphi d\,\mu\right| \leq M\ \|\varphi\|\ \rho$ for all simple functions φ. Then show that $g \in L^q$.

(c) State and prove Holder's inequality.

(d) State and prove Schwarz inequality.

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NUMBER THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Define multiplicative function. Prove that for $n,\ m \geq 1$

$$\phi\ (mn) \quad = \frac{\phi(m)\ \phi(n)}{\phi(d)} d\ if\ \ d(m,\mathrm{n})$$

$$= \phi(m)\ \phi(n)\ if\ \ (m\ n) = 1.$$

(b) State and prove Euler's summation formula

2. (a) Let P_n denote the n[th] prime. Then show that the following relations are equivalent

(i) $\lim\limits_{x \to \infty} \dfrac{\pi(x)\ log\ x}{x} = 1$

(ii) $\lim\limits_{x \to \infty} \dfrac{\pi(x)\ log\ (\pi(x))}{x} = 1$

(iii) $\lim\limits_{n \to \infty} \dfrac{\rho_n}{n\ log^{\rho n}} = 1$

(b) Prove that for every integer $n \geq 2$

3. (a) Let χ be any real valued character mod k and $A(n) = \sum\limits_{d/n} \chi\ (d)$. Then show that $A(n) \geq 0$ for all n and $A(n) \geq 1,$ if n is a square.

(b) For $n > 1$ and $\chi \neq \chi_1,$ prove that

$$\sum_{P \leq x} \frac{\chi(p) log\ p}{p} = -L^{-1}\ (1,\chi).\sum_{n \leq x} \frac{\mu(n)\chi(n)}{n} + 0(1)$$

4. (a) State and prove Gauss Lemma.

(b) For every odd prime p, prove that $\left(\frac{2}{p}\right) = (-1)^{\frac{(p^2-1)}{8}} = \begin{cases} 1, \text{ if } \ p \equiv \pm 1 \ (mod \ 8) \\ -1, \text{ if } \ p \equiv \pm 3 \ (mod \ 8) \end{cases}$

Section - B **(4x1=4)**

5. Answer all the Following :

(a) Define complete by multiplicative function. Give an example of a multiplicative function.

(b) Show that for $x > 1$,

$$\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} \ x^2 + 0(x \ log \ x)$$

(c) State and prove Euler - Fermet theorem

(d) If $m \ and \ n$ are odd positive integers then show that $\left(\frac{m}{p}\right)\left(\frac{n}{p}\right) = \left(\frac{mn}{p}\right)$.

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UNIVERSAL ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **4 x 4 = 16**

1. (a) Prove that L is non distributive lattice iff M_5 *or* N_5 can be embedded into L.

   (b) If for $\theta \in E_9(A)$ and $a, b \in A$ then show that

   (i) $A = \bigcup_{\alpha \in A} a/\theta$ (ii) $a/\theta \neq b/\theta$ implies $a/\theta \cap b/\theta = \phi$.

2. (a) If A is congruence permutable, then show that A is congruence modular.

   (b) Let $\alpha : A \to B$ be a homomorphism. Then prove that $Ker(\alpha)$ is a congruence on A.

3. (a) If for an indexed family of maps $\alpha_i : A \to A_i,\ i \in I,$ then show that the following are equivalent.

   (i) The maps x_i seperate points

   (ii) α is injective

   (iii) $\bigcap_{i \in I} Ker\,\alpha_i = \Delta$

   (b) Suppose $U_1(X_1)$ and $U_2(X_2)$ are two algebras in a class K with the universal mapping property for K with the universal mapping property for K over the indicated sets. $|X_1| = |X_2|$, then show that $U_1(X_1) \cong U_2(X_2)$.

4. (a) Let $B = \langle B, V, \wedge, \bullet, 0, 1\rangle$ be a Boolean algebra. Define $B^{\otimes}$ to be the algebra $\langle B, +, \bullet, -, 0, 1\rangle$, where $a + b = (a \wedge b') \vee (a' \wedge b)$ $a \wedge b = a \bullet b,$ $a' = 1 + a$. Then show that $B^{\otimes}$ is a Boolean ring.

   (b) Let B be a Boolean algebra.

   (i) Let F be a filter of B. Then show that F is an ultrafilter of B iff $0 \notin F$ and for $a, b \in B,$ $a \vee b \in F$ iff $a \in F$ or $b \in F$.

   (ii) Let I be an ideal of B. Then show that I is a maximal ideal of B iff $1 \notin I$ and for $a, b \in B,$ $a \wedge b \in I$ iff $a \in I$ *or* $b \in I$.

Section - B **(4 x 1 =4 Marks)**

5. Answer all the Following :

(a) Define the terms

(i) Lattice (ii) Partially ordered set.

(b) If $\alpha : A \to B$ is an embedding, then prove that $\alpha(A)$ is a subuniverse of B.

(c) If B is a Boolean algebra and $a, b \in B$ then show that $N_a \cup N_b = N_{a \vee b}$, $N_a \cap N_b = N_{a \wedge b}$ and $N_{a'} = (N_a)'$. Thus in particular the $N_a^{'}s$ form a basis for the topology of $B*$.

(d) Let X be a set. Then show that $SU(X) \cong 2^X$.

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COMMUTATIVE ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 1 =4 Marks)**

1. (a) Prove that the nil radical of A is the intersection of all prime ideals of A.

   (b) State and prove Nakayama's lemma.

2. (a) For any $A-$ module N prove the following.

   If $0 \to M' \to M \to M'' \to 0$ is any exact sequence of $A-$ modules, then the tensored sequence. $0 \to M' \otimes \mathrm{N} \to M \otimes \mathrm{N} \to M'' \otimes \mathrm{N} \to 0$ is exact.

   (b) If N, P are submodules of an $A-$ module M then prove that $S^{-1}(N+P) = S^{-1}(N) + S^{-1}(P)$ for any multiplicative subset S of A.

3. (a) State and prove first uniqueness theorem.

   (b) State and prove going down theorem

4. (a) State and prove Hilbert basis theorem.

   (b) In an Artin ring prove that the nil radical is nilpotent

Section - B **(4x1=4)**

5. **Answer all the Following :**

   (a) Prove that an $A-$ module M is finitely generated if and only if M is isomorphic to the quotient of A^n for some integer $n > 0$.

   (b) Let $A \subseteq B$ be rings, B integral over A. Then prove if that $x \in A$ is a unit in B then it is a unit in A.

   (c) Show that a module M has composition series if and only if M satisfies both chain conditions.

   (d) Prove that a ring A is Artin $\Leftrightarrow$ A is Noetherian and $dim\ A = (0)$.

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NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Write a program to evaluate double integral $\int_0^1 \left(\int_0^2 \frac{2xy\,dy}{(1+x^2)(1+y^2)} \right) dx$ using the Simpson's rule with $h = k = 0.25$.

   (b) Write a program to solve the ordinary differential equation $\frac{dy}{dx} - f(x, y)$ using Euler's method.

2. (a) Find the approximate value of the integral $I = \int_0^1 \frac{dx}{1+x}$ using

   (a) Composite trapezoidal rule with $2, 3, 5, 9$ nodes and Romberg integration.

   (b) Composite Simpson's rule with 3, 5, 9 nodes and Romberg integration.

Or

3. (a) Solve the initial value problem $u' = -2tu^2, \mathrm{u}(0) = 1$ with $h = 0.2$ on the interval $[0,1]$. Use the fourth order classical Runge - Kutta method.

   (b) Solve the boundary value problem $u'' = u + x, u(0) = 0, u(1) = 0$ with $h = \frac{1}{4},$ Use the numerov method.

4. (a) Write the program to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y},\ y(0,4) - 0.41$ with $h = 0.2$ by Runge - Kutta formula of fourth order.

   (b)

Section - B (4 x 1 = 4)

5. Answer all the Following :

(a) Explain the concepts of input out statement.

(b) Find the Jacobian matrix for the system of equations.

$$f_1(x, y) = x^2 + y^2 - x = 0.$$

$f_2(x, y) = x^2 - y^2 - y = 0$ at the point $(1, 1)$

(c) Explain the third order Runge - Kutta method.

(d) Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$ using the backward Euler method.

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(Optional P - II) LATTICE THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 1 =4 Marks)**

1. (a) Define partially ordered set and order isomorphism. Prove that two finite partially ordered sets can be represented by the same diagam if and only if they are order isomorphic.

   (b) Define a weakly complemented lattice and a semi complemented lattice. Prove that every uniquely complemented lattice is weakly complemented.

2. (a) State and prove lattice theoritical duality principle.

   (b) Let L be a distributive lattice. Then show that the lattice $\left(I\left(L\right), \leq\right)$ of all ideals of L is a distributive lattice.

3. (a) For a complete Boolean algebra B show that the following are equivalent.

   (i) B is completely meet-distributive

   (ii) B is completely join distributive

   (iii) B is atomic

   (iv) B is dually atomic

   (b) Let $(B, \wedge, \vee)$ be Boolean algebra. Then prove that $B\left(R\left(B\right)\right) = B$

4. (a) Show that in a Boolean algebra $B,$ a proper ideal M of B is prime if and only if it is maximal.

   (b) Prove that every ideal and dual ideal of a lattice L is a convex sub lattice of L. Conversely, prove that every convex sub lattice of L is the set intersection of an ideal and of a dual ideal.

Section - B

5. Answer all the Following :

(a) Show that any interval of a lattice is a sub lattice.

(b) Show that a modular lattice satisfies both the upper and lower covering conditions.

(c) Prove that any Boolean ring can be regarded as a Boolean algebra and Vice - Versa.

(d) If θ is a congruence relation on L then prove that every θ class is a convex sublattice of $L.$

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Optional - LINEAR PROGRAMMING AND GAME THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) State and prove fundamental duality theorem.

   (b) If c is a finite cone, then show that $c^{**}=c.$

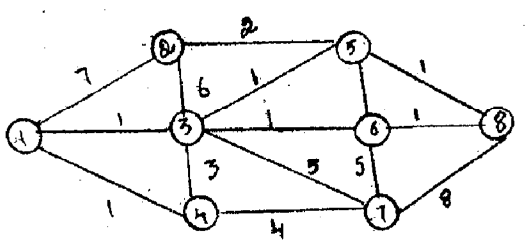
2. (a) If both a program and its dual are feasible then show that both have optimal vectors and the values of the two programs are the same. If either program is infeasible then neither has an optimal vector.

   (b) Find $y=(n_1,\ n_2,...,\ n_5)\geq 0$ which minimizes $n_1+6n_2-7n_3+n_4+5n_5$ subject to

   $5n_1-4n_2+13n_3-2n_4+n_5=20$, $n_1-n_2+5n_3-n_4+n_5=8$

3. (a) In the network below, nodes (1) and (6) are sources with supplies $\sigma(1)=3,\ \sigma(6)=5.$

   Nodes (4) and (8) are sinks with demands $\delta(4)=4,\ \delta(8)=4.$ The numbers on the edges are capacities which are assumed to be the same in both directions. Determine whether this transhipment problem is feasible.



   (b) Find a soultion of the optimal assignment problem whose rating matrix is the following.

0	15	9	1	3	4	19
2	0	19	11	9	3	14
12	5	17	12	24	15	16
19	11	14	23	16	17	29
20	15	23	22	19	21	24
23	12	16	17	24	25	26
25	16	8	26	21	20	23

4. (a) If ϕ and ϕ' are two functions of s and t with saddle values w and w' and for some $\in > 0$ and all (s,t). $|\phi(s,t) - \phi'(s,t)| \leq \in$, then show that $|w - w'| \leq \in$.

(b) Show that the game Γ has a solution if and only if $\hat{\Gamma}$ has a solution.

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

(a) Define

(i) convex set

(ii) convex hull

(iii) convex polytope.

(b) Write about the optimality criterion for simplex method.

(c) Define the terms

(i) Two person zero - sum game

(ii) Matrix game

(d) Show that a symmetric game Γ has a solution if and only if there exists $\bar{s}$ in S such that $\phi(s, \bar{s}) \geq 0$ for all s in S.

ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER 2018-2019

M.A. / M.Sc (Final) Mathematics

Optional - INTEGRAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Form an integral equation corresponding to the differential equation

$\left(d^2y/dx^2\right) - sin\ x\left(dy/dx\right) + e^x y = x,$ with the inital conditions

$y(0)=1,\ y'(0)=-1.$

(b) Solve $y'(t) = t + \int_0^1 y(t-x)\ cos\ xd\,x, \mathrm{y}(0) = 4$

2. (a) Solve the following symmetric integral equation with the help of Hilbert - Schmidt theorem.

$y(x) = 1 + \lambda \int_0^{\pi} cos\ (x+t)\, y(t)\, dt$

(b) Using Green's function, solve the boundary value problem $y'' - y = x,\ y(0) = y(1) = 0.$

3. (a) Find the resolvent Kernel of the Volterra integral equation with the Kernel $K(x,t) = (2+cos\ x)/(2+cos\ t).$

(b) Solve the integral equation $\int_{-\pi}^{\pi} \phi(y) log\ |cos\ x - cos\ y|\, dy = f(x),\ -\pi \le x \le \pi$

4. (a) Solve $y(x) = cos\, x - x - 2 + \int_0^x (t-x)\ y(t)\ dt$

(b) Find the eigen values and eigen functions of the homogeneous integral equation

$$y(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt} \right) y(t)\, dt$$

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

(a) Define Fredholm integral equations of the first and second kind.

(b) Write the four properties to construct the Green's functions.

(c) Solve the Volterra integral equation of the first kind $\int_0^1 y(x)\, y(t-x)\,dx = 16\ sin\ 4t.$

(d) Using the method of successive approximation, solve the integral equation $y(x) = 1 + x - \int_0^x y(t)\, dt,$ taking $y_0(x) = 1.$