ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER 2021-2022

M.A. / M.Sc. Mathematics (Previous)

ALGEBRA

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) State and prove Lagrange's theorem.

   (b) State and prove Cayley's theorem.

2. (a) Show that alternating group A_n is generated by the set of all 3-cycles in S_n.

   (b) Show that the alternating group A_n is simple if $n > 4$.

3. (a) Let $f : R \to S$ be a homomorphism of a ring R into a ring S. Then prove that $Ker\ f = \{0\}$ if and only if f is 1-1.

   (b) If R is a commutative ring, then prove that an ideal P in R is prime if and only if $ab \in P$, $a \in P, b \in R$ implies $a \in P\ or\ b \in P$.

4. (a) State and prove Eisentein criterion.

   (b) Let $P(x)$ be an irreducible polynomial in F[x] then show that there exists an extension E of F in which $P(x)$ has a root.

Section – B **(4 x 1 = 4)**

5. Answer all the following :

   (a) Show that center of a group is a normal subgroup of G.

   (b) Prove that the centre of a ring is a subring.

   (c) Show that every Euclidean domain is a PID.

   (d) Find the smallest extension of Q having a root of $x^2 + 4 \in Q[x]$.

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LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Let T be a linear operator on an n - dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

   (b) State and prove Cayley's theorem.

2. (a) Let $y_1(x) and\ y_2(x)$ be two solutions of the second order differential equation $y'' + P(x)y' + Q(x)y = 0\ on\ [a, b]$. Then prove that their Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on [a,b].

   (b) Find the general solution of $y'' - 2y' = 12x - 10$ by the method of undetermined coefficients.

3. (a) Solve the system
   $\frac{dx}{dt} = 3x - 4y\ ,\ and\ \frac{dy}{dt} = x - y.$

   (b) State and prove Picard's theorem.

4. (a) By the method Laplace transforms, find the solution of $y'' - 4y' + 4y = 0, y(0) = 0\ and\ y'(0) = 3.$

   (b) State and prove convolution theorem on Laplace transforms.

Section - B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) Define similar matrices and prove that similar matrices will have the same characteristic polynomial.

   (b) Find the inverse laplace transforms of $\frac{12}{(p+3)^4}$.

   (c) What are the Voltera's pre-predator equations? Describe the dynamic behavior of these equations.

   (d) Find the general solution of $2y'' + 2y' + 3y = 0.$

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REAL ANALYSIS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) If $\bar{E}$ is the closure of a set E in a metric space X, then prove that $diam\, \bar{E} = diam\, E$.

   (b) Suppose f is a continuous real function on a compact metric space X and $M = \operatorname{Sup}_{p \in X} f(p)$, $m = \operatorname{Inf}_{p \in X} f(p)$. Then prove that there exist points $p, q \in X$ such that $f(p) = M$ and $f(q) = m$.

2. (a) State and prove necessary and sufficient condition for the existence of Riemann-Stieltjes integral.

   (b) State and prove fundamental theorem of integral calculus.

3. (a) State and prove Cauchy criterion for uniform convergence of sequence of functions.

   (b) State and prove Stone's generalization of the Weierstrass theorem.

4. (a) If X is a complete metric space, and if φ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

   (b) State and prove inverse function theorem.

Section – B **(4 x 1 = 4)**

5. **Answer all the following :**

   (a) If $0 \le x < 1$, then prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

   (b) State and prove integration by parts formula.

   (c) Give an example of a convergent series of continuous functions that may have a discontinuous sum.

   (d) State and prove linear inversion of implicit function theorem.

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M.A. / M.Sc. Mathematics (Previous)
TOPOLOGY

Note: Answer ALL Questions.
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Section - A **(4 x 4 =16 Marks)**

1. (a) Let X be a metric space then prove that a subset G of X is open if and only if it is union of open spheres.

   (b) Let X and Y be metric spaces and $f : X \rightarrow Y$ a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

2. (a) Define (i) Base and (ii) subbase for a topology on non-empty set (iii) Prove that the set of all open intervals in R forms a sub base for the standard topology of R and also prove that the set of all open rays from a base for the standard topology of R.

   (b) State and prove Heine - Borel theorem.

3. (a) Prove that every compact Hausdorff space is normal.

   (b) State and prove Ascoli's theorem.

4. (a) Prove that product of Hausdorff spaces is Hausdorff.

   (b) State and prove Weierstrass approximation theorem.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let X be an arbitrary non-empty set, and define d by $d(x,y) = \begin{cases} 0, & x = y \\ 1, & x \neq y. \end{cases}$

Then prove is a metric on X.

(b) Show that a subspace of a topological space is itself a topological space.

(c) Prove that Continuous image of connected set is connected.

(d) Prove that every compact metric space is complete and totally bounded.

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DISCRETE MATHEMATICS

Note: Answer ALL Questions.
All Questions carry equal marks.

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.
   (b) Prove that a graph and its complement cannot both be disconnected.

2. (a) Prove that there are $\frac{n+1}{2}$ pendant verticles in any binary tree with n vertices.
   (b) Prove that a graph G is Eulerian if and only if every vertex of G is of even degree.

3. (a) Describe an automaton and semi automaton. Describe the cafeteria a automaton and draw the state graph of this automaton.
   (b) Explain by means of an example the concept of an automaton associated with a monoid (S, ·). Show that there exists an automaton whose monoid is isomorphic to (S,·).

4. (a) State and prove DeMorgan's Laws in a Boolean algebra.
   (b) Prove that the cardinality of a finite Boolean algebra is always of the form 2^n and any two Boolean algebras with the same cardinality are isomorphic.

Section – B **(4 x 1 = 4)**

5. **Answer all the following :**
   (a) Show that a graph is a tree if and only if it has no cycles and |E| = |V|-1.
   (b) Show that a linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.
   (c) Define that every distributive lattice is modular.
   (d) Prove that the distance of a linear code is equal to the minimum weight of any non zero codewords.