

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Previous) Mathematics

ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) State and prove Lagrange's theorem.
(b) Let G be a group and G' be the derived group of G . Then show that the following hold
 - (i) G' is normal subgroup of G
 - (ii) The quotient group G/G' is abelian
 - (iii) If $H \leq G$, then G/H is abelian if and only if $G' \subseteq H$.
2. (a) Prove that for any n , the alternating group A_n is generated by 3-cycles.
(b) Prove that for any $n > 4$, the alternating group A_n is simple.
3. (a) Show that every finite integral domain is a division ring.
(b) State and prove fundamental theorem of homomorphism of rings.
4. (a) State and prove Eisenstein criterion
(b) Let $f(x) \in \mathbb{Z}[x]$ be a primitive polynomial then show that $f(x)$ is reducible over \mathbb{Q} if and only if $f(x)$ is reducible over \mathbb{Z} .

Section - B

(4 x 1 = 4)

5. Answer all the following :

- (a) Prove that any subgroup of a cyclic group is cyclic.
- (b) Show that the set of all even permutations A_n in S_n is of index 2 in S_n .
- (c) Show that if R is a ring with unity, then each maximal ideal is prime, but the converse, in general is not true.
- (d) Find the minimal polynomial of $2 + \sqrt{5}$ over \mathbb{Q} .

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ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Previous) Mathematics

REAL ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 =16 Marks)

1. (a) If X is a compact metri space and if $\{p_n\}$ is a Cauchy sequence in X then prove that $\{p_n\}$ converges to some point of X .

(b) State and prove Merten's theorem.

2. (a) Suppose $C_n \geq 0$ for $n=1, 2, 3, \dots$, $\sum C_n$ converges, $\{s_n\}$ is a sequences of distinct points in (a, b) and $a(x) = \sum_{n=1}^{\infty} C_n I(x - s_n)$. Where I is the unit step function. Let

f be continuous on $[a, b]$. Then prove that $\int_a^b f d\tau = \sum_{n=1}^{\infty} C_n f(s_n)$.

(b) Assume that τ increase monotonically and $\tau' \in \mathfrak{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Prove that $f \in \mathfrak{R}(\tau)$ if and only if $f\tau' \in \mathfrak{R}$. In that case,

prove that $\int_a^b f d\tau = \int_a^b f(x)\tau'(x)dx$.

3. (a) Let τ be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathfrak{R}(\tau)$ on $[a, b]$ for $n=1, 2, 3, \dots$, and suppose $f_n \rightarrow f$ uniformly on $[a, b]$. Then prove that $f \in \mathfrak{R}(\tau)$

on $[a, b]$ and $\int_a^b f d\tau = \lim_{n \rightarrow \infty} \int_a^b f_n d\tau$.

(b) If K is compact, if $f_n \in \mathfrak{C}(K)$ for $n=1, 2, 3, \dots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K , then prove that

(i) $\{f_n\}$ is uniformly bounded on K , and

(ii) $\{f_n\}$ contains a uniformly convergent subsequence.

4. (a) Suppose \bar{f} maps a convex open set $E \subset \mathbb{R}^n$ into \mathbb{R}^m , \bar{f} is differentiable in E , and there is a real number M such that $\|f^{-1}(x)\| \leq M$ for every \bar{x} in E . Then prove that $\|\bar{f}(\bar{b}) - \bar{f}(\bar{a})\| \leq M \|\bar{b} - \bar{a}\|$ for all \bar{a} in E , \bar{b} in E .
- (b) State and prove that contraction principle.

Section - B

(4x1=4)

5. Answer all the Following :

- (a) Give an example to show that the product of two convergent series may actually diverge.
- (b) If $f \in \mathfrak{R}(r)$ and $g \in \mathfrak{R}(r)$ then prove that

(i) $fg \in \mathfrak{R}(r)$ and

(ii) $|f| \in \mathfrak{R}(r)$ and $\left| \int_a^b f \, dr \right| \leq \int_a^b |f| \, dr$

- (c) Give an example to show that a convergent series of continuous functions may have a discontinuous sum.
- (d) State and prove the linear version of the implicit function theorem.

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ASSIGNMENT QUESTION PAPER 2017-2018
M.A. / M.Sc (Previous) Mathematics
TOPOLOGY

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Let X be a metric space. Prove that subset F of X is closed if and only if its complement F' is open.

(b) Let X be a complete metric space and let Y be a subspace of X . Prove that Y is complete if and only if it is closed.
2. (a) Prove that every separable metric space is second countable.

(b) State and prove the Heine - Borel theorem.
3. (a) State and prove the Tietze Extension theorem.

(b) State and prove the Urysohn Imbedding theorem.
4. (a) Prove that $[a, b]$ is separable.

(b) Let X be an arbitrary topological space. Prove that every closed subalgebra of $C(X, R)$ is also a closed sublattice of $C(X, R)$.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) In the discrete metric space, prove that singleton sets are open. Deduce that each subset of a discrete metric space is both open and closed.
- (b) Show that any continuous image of a compact space is compact.
- (c) Prove that every compact subspace of a Hausdorff space is closed.
- (d) Show that any continuous image of a connected space is connected.

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ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Previous) Mathematics

DISCRETE MATHEMATICS

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Define complement of a graph. Prove that any graph and its complement cannot both be disconnected.
 (b) Prove that any tree with n vertices has $n-1$ edges.
2. (a) Prove that a lattice L is modular if no sublattice of L is isomorphic to N_5 .
 (b) Give an example of a modular lattice but not distributive with details
3. Minimize the following automata.

State	u		}	
	a_1	a_2	a_1	a_2
z_1	z_1	z_2	0	0
z_2	z_2	z_3	0	0
z_3	z_2	z_3	0	0
z_4	z_3	z_1	0	0
z_5	z_2	z_1	0	0
z_6	z_4	z_5	0	1
z_7	z_6	z_7	0	1

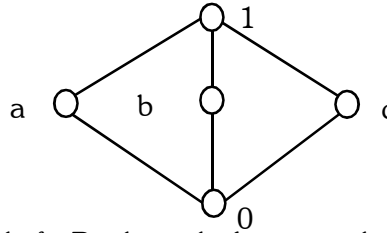
4. (a) Let $C = \{110010, 101111, 111011, 000111, 111111\}$ be the code. Find the distance of C . Find all error patterns that can be detected by C .
 (b) Prove that the distance of a linear code is equal to the least of the weights of all non-zero code words.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) Define connected graph and give an example of a disconnected graph.
- (b) Prove that the lattice with the following Hasse diagram is a modular



- (c) Let M be an ideal of a Boolean algebra B such that for each $a \in B$, either $a \in M$ or $a' \in M$ but not both hold. Prove that M is a maximal ideal of B .
- (d) Let C be a linear code with generating matrix $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$. Write all words in C .

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER 2017-2018
M.A. / M.Sc (Previous) Mathematics
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Show that every n -dimensional vector space over the field F isomorphic to the space F^n .
- (b) Show that $T \in L(V)$ is triangulable if and only if its minimal polynomial $p(x)$ has the form $p(x) = (x - \lambda_1)^{r_1} \dots (x - \lambda_k)^{r_k}$ for $0 \leq r_i \leq k, i \in \{1, 2, \dots, k\}$
2. (a) Solve $y'' + k^2y = 0$
- (b) Show that if $y_1(x)$ and $y_2(x)$ are two solutions of $y'' + p(x)y' + Q(x)y = 0$ then $c_1y_1(x) + c_2y_2(x)$ is also a solution for any constants c_1 and c_2 and show that if y_1 and y_2 have a common zero in the interval $[a, b]$, then one is constant multiple of the other.

3. (a) Solve the system
$$\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$$

(b) If two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}, \begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the system
$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$$
 have

Wronskian $w(t)$ that does not vanish on $[a, b]$ then prove that
$$\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$$
 is the general solution of the given system.

4. (a) Compute $L(f)$ for $f(x) = \sin ax \forall x \in [0, \infty)$.
- (b) Solve the following second order linear differential equation by Applying Laplace transform:
- (i) $y'' - 4y' + 4y = 0$, with initial conditions $y(0) = 0$ and $y'(0) = 3$.
- (ii) $y'' + y' = 3x^2$ and initial conditions $y(0) = 0$ and $y'(0) = 1$.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) Let N be the Nilpotent operator given by the matrix $B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ w.r.t. standard basis. Find the canonical form of N .
- (b) Find the particular solution of the differential equation $y' = xe^x$ which satisfies the initial conditions $y = 3$ when $x = 1$.
- (c) Find the general solution of the equation $y'' + y = 0$ from the given one solution $y_1(x) = \sin x$.
- (d) Find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = 4x + 5y \end{cases}$$

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Final) Mathematics
COMPLEX ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 =16 Marks)

1. (a) Let u and v be real valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Prove that $f:G \rightarrow \mathbb{C}$ defined by $f(z) = u(z) + iv(z)$ is analytic iff u and v satisfy the Cauchy - Reimann equations.
- (b) Let Z_1, Z_2, Z_3, Z_4 be four distinct points in \mathbb{C}_∞ . Prove that (Z_1, Z_2, Z_3, Z_4) is a real number iff all four points lie on a circle.
2. (a) State and prove Rouché's Theorem.
- (b) State and prove Schwarz's Lemma.
3. (a) Prove that a set of $\subset C(G, \Omega)$ is normal iff its closure is compact.
- (b) State and prove the Montel's Theorem.
4. (a) Explain the notion of an analytic function along a given path.
- (b) Define an harmonic function with the usual notation that if u is harmonic then show that $f = u_x - iu_y$ is analytic.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) If $r: [0, 1] \rightarrow \mathbb{C}$ is closed rectifiable curve and $a \notin \{r\}$, then prove that $\frac{1}{2\pi i} \int \frac{dz}{z-a}$ is an integer.
- (b) Classify the singularities of (i) $\cot z$ (ii) $e^{1/z}$ (iii) $\frac{z+1}{(z^2+1)^2}$
- (c) Prove that $(C(G, \Omega), \dots)$ is a metric space.
- (d) Prove that the usual notation that if u is harmonic then $u_x = \frac{\partial u}{\partial x}$ and $u_y = \frac{\partial u}{\partial y}$ are also harmonic.

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ASSIGNMENT QUESTION PAPER 2017-2018
M.A. / M.Sc (Final) Mathematics
MEASURE THEORY AND FUNCTIONAL ANALYSIS

Answer any ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that

$$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^*(A_n).$$

- (b) Let E be a measurable set of finite measure, and $\{f_n\}$ a sequence of measurable functions defined on E . Let f be a real valued function such that for each x in E we have $f_n(x) \rightarrow f(x)$. Then prove that given $\epsilon > 0$, and $u > 0$, there is a measurable set $A \subset E$ with $m(A) < u$ and an integer N such that for all $x \notin A$ and all $n \geq N$, we have $|f_n(x) - f(x)| < \epsilon$.

2. (a) Let \sim be a measure on a σ -algebra B , If $E_i \in B$, $\sim(E_1) < \infty$ and $E_i \supset E_{i+1}$ for

$$i = 1, 2, 3, \dots \text{ then prove that } \sim\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \rightarrow \infty} \sim(E_n).$$

- (b) Let χ be a signed measure on the measurable space (X, B) . Let E be a measurable set such that $0 < \chi(E) < \infty$. Then prove that there is a measurable set A contained in E with $\chi(A) > 0$.

3. (a) If N and N^1 are normed linear spaces then prove that the set $B(N, N^1)$ of all continuous linear transformations of N into N^1 is itself a normed linear space with respect to the pointwise linear operations and the norm defined by

$$\|T\| = \sup\{\|T(x)\| : \|x\| \leq 1\} \text{ for } T \in B(N, N^1). \text{ Further, if } N^1 \text{ is a Banach space then}$$

prove that $B(N, N^1)$ is also a Banach space.

- (b) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M , then prove that there exists a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
4. (a) State and prove Schwarz inequality.
- (b) If M is a closed linear subspace of Hilbert space H then prove that $H = M \oplus M^\perp$.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) Show that if E_1 and E_2 are measurable then
- $$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$
- (b) For $g \in L^q$ let F be the linear functional on \underline{P} defined by $F(f) = \int fg \, d\mu$. Show that $\|F\| = \|g\|_q$.
- (c) If P is a projection on a Banach space B , and if M and N are its range and null space, then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.
- (d) If P is a projection on a Hilbert space H with range M and null space N , then prove that $M \perp N$ if and only if P is self - adjoint; and in this case, $N = M^\perp$.

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ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Final) Mathematics
NUMBER THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 =16 Marks)

1. (a) For $n \geq 1$, prove that $\sum_{d|n} W(d) = n$.
- (b) Prove that $W(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.
- (c) State and prove mobius inversion formula.

2. (a) Let $x > 0$. Prove that $0 < \frac{\Psi(x)}{x} - \frac{\pi(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x} \log 2}$ Ψ, π are Chebyshev's functions.
- (b) For $n \geq 1$, prove that $2^n \leq 2^n c_n < 4^n$

3. (a) Let G be a finite abelian group with elements a_1, a_2, \dots, a_n and f_1, f_2, \dots, f_n the characters of G . Then prove that

$$\sum_{r=1}^n \overline{f_r(a_i)} f_r(a_j) = \begin{cases} n & \text{if } a_i = a_j \\ 0 & \text{if } a_i \neq a_j \end{cases}$$
- (b) State and prove Dirichlet theorem for infinitely many primes in an arithmetical progression (using necessary lemmas)

4. (a) State and prove quadratic reciprocity law.
- (b) Let p be an odd prime with $\left(\frac{p}{n}\right)$

Then prove that $G(n:p) = \left(\frac{n}{p}\right) G(1:p)$ where $G(n:p)$ is quadratic Gauss sum.

Section - B

(4x1=4)

5. Answer all the Following :

- (a) If $x \geq 1$ prove that $\sum_{n>x} \frac{1}{n^s} = O(x^{1-s})$ for $s > 1$.
- (b) Let a, m be integers such that $(a, m) = 1$. Then prove that the linear congruence $ax \equiv b \pmod{m}$ has exactly one solution.
- (c) Solve the congruence $5x \equiv 3 \pmod{24}$
- (d) If p and w are odd positive integers then show that $\binom{n/p}{p} \binom{m/p}{p} = \binom{mn/p}{p}$.

ANDHRA UNIVERSITY
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M.A. / M.Sc (Final) Mathematics

UNIVERSAL ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A

4 x 4 = 16

1. (a) Prove that two lattices L_1 and L_2 are isomorphic iff there is a bijection Γ from L_1 to L_2 such that both Γ and Γ^{-1} are order - preserving.
(b) Prove that L is non distributive lattice iff M_s or N_s can be embedded into L .
2. (a) Let A be an algebra and define $Sg(x) = n\{B/x \subseteq B\}$ and B is a subuniverse of A . Then prove that Sg is an algebraic closure operator on A .
(b) Let A be an algebra all of whose fundamental operations have arity at most n . Then prove that S_g is an n -ary closure operator on A .
3. (a) If \sim, \sim^* is a pair of factor congruences on an algebra A , then prove that $A \cong A/\sim \times A/\sim^*$ under the map $\Gamma(a) = (\Gamma/\sim, \Gamma/\sim^*)$.
(b) Prove that an algebra A is sub directly irreducible iff A is trivial on there is a minimum congruence in $con(A) - \{\Delta\}$.
4. (a) Let X be a set. Then prove that $S_\sim(x) \cong 2^x$.
(b) Prove that every finite Boolean algebra is isomorphic to the Boolean algebra of all subsets of some finite set X .

Section - B

(4 x 1 =4 Marks)

5. Answer all the Following :

- (a) Define the term lattice and give two examples of a lattice
- (b) Define the terms (i) Boolean algebra (ii) Heyting algebra
- (c) Define the terms (i) Factor congruence (ii) indecomposable algebra.
- (d) In a Boolean algebra, prove that (i) $(x \vee y)' = x' \wedge y'$ (ii) $(x \wedge y)' = x' \vee y'$

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ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Final) Mathematics
COMMUTATIVE ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 1 =4 Marks)

1. (a) Let I be an ideal of a ring A . Then prove that there is a one to one corresponding between the ideals of A containing I and the ideals of A/I .
- (b) Define the radical of an ideal in a ring and prove that the radical of an ideal I of A is the intersection of all prime ideals of A containing I .
2. (a) Let M and N be A -modules. Then prove that the tensor product of M and N exists.
- (b) Let M, N, P be A -modules. Then prove that there exist unique isomorphisms.
 - (i) Between $M \otimes N$ and $N \otimes M$
 - (ii) $(M \otimes N) \otimes P$ and $M \otimes (N \otimes P)$
3. (a) State and prove first uniqueness theorem for primary decomposition
- (b) Let A be a subring of a ring B and $x \in B$. Then prove that the following are equivalent.
 - (i) x is integral over A .
 - (ii) $A[x]$ is finitely generated A -module.
 - (iii) $A[x]$ is contained in a subring C of B such that C is finitely generated A -module.
 - (iv) There exists a faithful $A[x]$ -module M which is finitely generated as an A -module.

4. (a) State and prove Hilbert basis theorem.
(b) In a Noetherian ring prove that every ideal is a finite intersection of primary ideals.

Section - B

(4x1=4)

5. Answer all the Following :

- (a) Let I_1, I_2 be ideals of a ring A and P be a prime ideal of A such that $P \supseteq I_1 \cap I_2$.
Then $P \supseteq I_1$ or $P \supseteq I_2$
- (b) For A – submodules M and N , prove that $Ann(M + N) = Ann(M) \cap Ann(N)$.
- (c) Let B be an integral domain, K field of fractions of B . Then prove that B is a local ring.
- (d) In an artinian ring prove that the nil radical is equal to Jacobson radical.

ANDHRA UNIVERSITY
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ASSIGNMENT QUESTION PAPER 2017-2018
M.A. / M.Sc (Final) Mathematics
NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES
Answer ALL Questions
All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Write down the FORTRAN expressions corresponding to

(i) $\frac{\dagger + \sqrt{2} \sin \bar{r}}{a^2 + b^2 \dagger \cos r}$ (ii) $2.5 \log_{10} x + |x^2 - y^2| + \cos 42^0$. (iii) $\frac{1 - e^{-|x|}}{1 + e^{|x|}}$

- (b) If $A = 4.0, B = 3.0, C = 5.0, D = 5.0, E = 6.0$ then find the order of evaluation and the value of the expression $((A + B) * C ** 1.5 - D + E - A) / B + C$.

2. (a) Given the values of x and $y(x)$, find $y'(0.4)$ and $y''(1.0)$.

x : 0.4 0.6 0.8 1.0

$y(x)$: 0.672944 0.9431 1.1755 1.3863

- (b) Evaluate $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin x} dx$ using composite Simpson's rule with 6 sub - intervals.

Or

3. (a) Evaluate $\int_{-1}^1 e^{-x^2} \sqrt{1-x^2} dx$ using Gauss- Legendre 1- point, 2-point and 3-point formula.

- (b) Evaluate $\int_0^1 e^{-x} dx$ using trapezoidal rule with 1, 2, 4 and 8 sub-intervals. Improve your result by Romberg method.

4. (a) Given $\frac{dy}{dx} = y^2 + x^2, y(1) = 1$, compute $y(1.2)$ using Runge - Kutta method of order 4 by taking $h = 0.1$.

- (b) Solve the boundary value problem $u'' = u + 1, 0 < x < 1, u(0) = 0, u(1) = e - 1$, using the shooting method. Use Euler - Cauchy method with $h = 0.25$ to solve the initial

value problems. Also compare the solution with the exact solution $u(x) = e^x - 1$.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

(a) Write a function subprogram to calculate the factorial of a given number.

(b) Evaluate $f'(1)$ from the following data :

$x:$	1	1.1	1.2	1.3
$f(x):$	4	6	8	32

(c) Derive the general quadrature formula for numerical integration.

(d) Explain backward Euler method.

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Final) Mathematics

(Optional P - II) LATTICE THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 1 =4 Marks)

1. (a) Prove that every finite partial ordered set can be represented by a diagram.
(b) If a partly ordered set P satisfies the minimum condition, then prove that to any $n \in P$, there exists at least one minimal element m of P such that $n \geq m$.
2. (a) If P is a partly ordered set bounded above each of whose non-void subset R has an infimum, then prove that each non-void subset of P will have supremum and hence P is a complete lattice.
(b) Prove that every order preserving mapping of a complete lattice into itself has a finite element.
3. (a) Prove that the set $R(\sim)$ of all relations definable on a set M form a complete Boolean algebra with respect to the ordering
$$W \leq E \Leftrightarrow ((x, y) \in W \text{ implies } (x, y) \in E \text{ (} x, y \in \sim \text{)}) \text{ for all } W, E \in R(M).$$

(b) Let \leq be any Boolean sub algebra of a Boolean $\dagger -$ algebra A . Then prove that the set \sim of all measurable elements of A is a Boolean sub algebra of A including S .
4. (a) The congruence lattice of any algebra is compactly generated.
(b) Prove that every ideal of a lattice L be the Kernel of at most one congruence relation of L , it is necessary and sufficient that L is distributive.

Section - B

5. Answer all the Following :

- (a) Show that if $x_1 \leq x_2 \leq \dots \leq x_n \leq x_1$ in a poset then $x_1 = x_2 = \dots = x_n$.
- (b) Prove that every chain is a modular lattice
- (c) In a Boolean algebra, prove that $a \leq b \Leftrightarrow b' \leq a'$
- (d) Prove that the set union of any ideal chain of a lattice L is itself an ideal in L .

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ASSIGNMENT QUESTION PAPER 2017-2018
M.A. / M.Sc (Final) Mathematics
Optional - LINEAR PROGRAMMING AND GAME THEORY
Answer ALL Questions
All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) If C is a finite cone in \mathfrak{R}^m , prove that $C^{**} = C$.
 (b) If X is the set of extreme points of a convex polytope K , prove that $k = \langle X \rangle$.
2. State and prove the fundamental duality theorem.
3. Solve the following transportation problem

	Markets				
	M_1	M_2	M_3	M_4	Supply \dagger_i
P_1	3	3	8	2	3
P_2	2	4	7	7	5
P_3	1	5	4	6	7
Demand	2	5	4	4	

4. (a) State and prove mini max theorem.
 (b) Solve the following game problem by linear programming method

$$\begin{bmatrix} 4 & 1 & -3 \\ 3 & 1 & 6 \\ -3 & 4 & -2 \end{bmatrix}$$

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) Write the dual to the following L.P.P.

$$\text{Maximize } Z = 2z_1 + z_2 + 3z_3$$

Subject to

$$z_1 - z_2 + 2z_3 \leq 3$$

$$2z_1 + 2z_2 - z_3 \leq 2$$

$$z_i \geq 0 \quad (i = 1, 2, 3)$$

- (b) Find the inverse of the following matrix using the replacement operation

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix}$$

- (c) Explain the concept of the simple assignment problem.

- (d) Solve the following game $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER 2017-2018

M.A. / M.Sc (Final) Mathematics
Optional - INTEGRAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A

(4 x 4 = 16 Marks)

1. (a) Suppose that an elastic string is fixed at two distinct end points. If the weight is attached between these points, find the integral representation for the displacement of the string due to the complete weight distribution, assuming that the tension in the string is uniform. Further, find the weight distribution function in terms of the displacement of the string.
- (b) Form the integral equation corresponding to the differential equation $\frac{d^3 y}{dx^3} - 2xy = 0$, with the initial conditions $y(0) = \frac{1}{2}$, $y'(0) = 1 = y''(0)$.
2. (a) Solve $\int_{-1}^{+1} (6x^2 + 4xy^2) \{ (y) dy = 3x^2 + 4x$
- (b) Prove that if K is Hermitian then the eigen functions corresponding to different eigen values of the problem $\{ (x) = \} \int_a^b K(x, y) \{ (y) dy$ are orthogonal.
3. (a) Find the resolvent kernel of $\{ (x) = \} \int_0^x e^{K(x-y)} \{ (y) dy + f(x)$
- (b) Solve : $\int_0^x \sin \Gamma (x-y) \{ (y) dy = 1 - \cos(Sx)$, Γ, S are given reals, by the method of Laplace transforms.
4. (a) Write the Picard's method for the existence of solutions of nonlinear Volterra integral equation of second kind $\{ (x) = f(x) + \} \int_0^x F(x, y, \{ (y)) dy$
- (b) Find the first and second approximation in the iterative solution of the integral equation. $\int_0^1 (x+y)^{\frac{1}{2}} [\{ (x)]^{\frac{1}{2}} dy = \{ (x)$ and find bounds on $\{ (x)$.

Section - B

(4 x 1 = 4)

5. Answer all the Following :

- (a) Describe the shop stocking problem
- (b) State Hilbert Schmidt theorem
- (c) Solve Abel's equation

$$\int_0^x (x-y)^{-r} \phi(y) dy = f(x), 0 < r < 1.$$

- (d) Use Galerkin's method to obtain an approximate solution ϕ of the form $\phi(x) = a_0 + a_1x$ for the integral equation

$$\phi(x) = x + \int_0^1 K(x, y) \phi(y) dy, \text{ where}$$

$$K(x, y) = \begin{cases} x(1-y), & x \leq y \\ y(1-x), & x \geq y \end{cases}$$